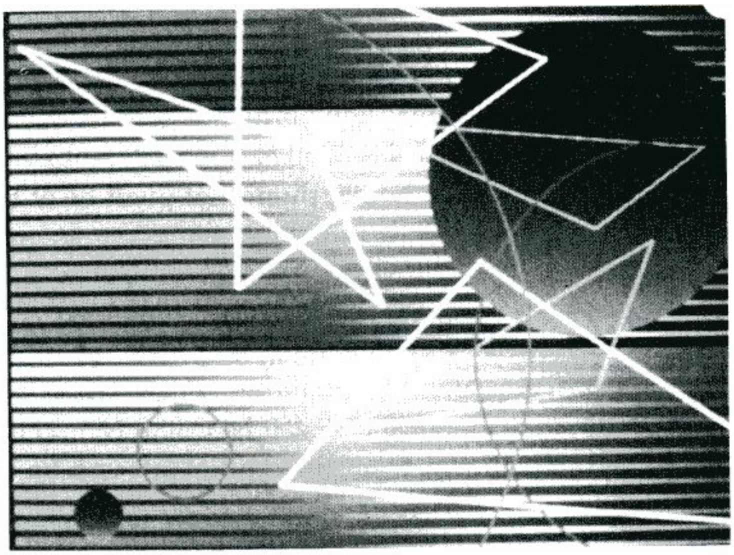
4 Concurrent and Parallel Forces



force (FŌRS) *n.: a physical quantity that can affect the motion of an object.*

OBJECTIVES

- Identify forces as vectors.
- Define and calculate resultant and equilibrant forces.
- Resolve forces into components.
- Define and identify frictional forces.
- Solve problems involving frictional forces.
- Define and calculate torque.
- Solve motion problems by applying the two conditions of equilibrium.

COMPOSITION OF FORCES

4.1 Describing Forces In Chapter 3, we studied the relationship between forces and motion. Now we shall take a look at some other characteristics of forces, and see how these characteristics are used in solving force problems.

When you push a door shut with your hand, your hand exerts a force on the door. The door also exerts a force on your hand. When you sit in a chair, you push on the chair and the chair pushes on you. In both of these cases, there is physical contact between the objects that are exerting forces on each other.

Forces can also be exerted without such physical contact. While an object is falling toward the earth, the earth exerts a gravitational force on the object and the object exerts a gravitational force on the earth. Yet there is no physical contact between the earth and the falling object.

These examples illustrate several important characteristics of forces:

1. *A net force will change the state of motion of an object.* The door moves because the force exerted by your hand is sufficient to overcome friction and other forces acting on the door. An object falls because a force is pulling it toward the earth. As we saw in Chapter 3, the application of a net force to an object always produces an acceleration.

2. *Forces can be exerted through long distances.* Gravitational and magnetic forces have this characteristic.

A force never exists by itself.

3. *Forces always occur in pairs.* When one object pushes

or pulls on another object, there is a force on *each* of the two objects. In the given examples, the two objects were your hand and the door, you and the chair, and the falling object and the earth.

4. *In each pair of forces, the two forces act in exactly opposite directions.* You pushed on the door, and the door pushed back. You pushed down, and the chair pushed up. The earth pulled the falling object toward the earth's center, and the object pulled the earth toward the object's center.

Now let us see how the magnitudes of forces are measured. (You will remember from Chapter 3 that forces are vector quantities and have both magnitude and direction.) When an object is suspended from a spring, it is pulled toward the earth by the force of gravitation. The spring stretches until the restoring force of the spring is equal to the force of gravitation on the object. Another object having the same weight stretches the spring by the same amount. Both objects together stretch the spring twice as far, and an object with three times the weight stretches it three times as far, etc. This characteristic of coiled springs, that the amount of stretch is proportional to the force pulling on the spring, means that we can use the amount of stretch to measure the size of a force. A device that measures forces in this way is called a *spring balance*. The results of such measurements can always be expressed in terms of newtons.

Figure 4-1. This telescoping boom crane can lift weights of more than 17 000 newtons to a height of sixteen stories.

Refer to Section 1.8 for a discussion of spring and platform balances.

4.2 Combining Force Vectors

Since forces have magnitude and direction, and can combine like displacements, they are vector quantities. When a vector is used to represent a force, the magnitude of the force is represented by the length of the arrow. The direction of the force can be deduced from the physical situation. For example, suppose a barge is being towed through still water by a tugboat. The tugboat applies a force of 10 $\bar{0}$00 N to the barge through the towline. The length of the arrow representing the force is proportional to the magnitude of the force, 10 $\bar{0}$00 N. Figure 4-2 is a diagram of this example. The point of application of the force is the point at which the rope is attached to the barge. A long rope can transmit only a pull in a direction along its length. It cannot transmit a push or a sideways force. The rope is in the direction of the force. The arrow shows this direction.

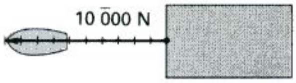


Figure 4-2. A vector diagram of a force of 10 $\bar{0}$00 N applied by a tugboat to a barge through a towline. The direction and point of application of the vector represent the line of action and point of application of the force.

Force vectors are treated like velocity vectors. For example, suppose two tugboats are attached to the same barge. Tugboat **A** is pulling with a force of 10 $\bar{0}$00 N and tugboat **B** is pulling with a force of 7500 N in the same direction.

Figure 4-3 represents this situation with vectors. The resultant force vector is 17 500 N in the direction of the towline. Figure 4-4 shows a vector diagram representing two tugboats pulling in opposite directions on a barge. Even though the forces act on extended objects such as opposite ends of the barge, the vectors may be considered as acting at the same point. The resultant force vector is 2500 N in the direction of tugboat **A**.

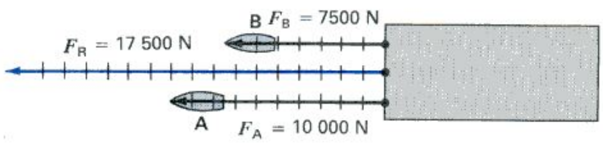


Figure 4-3. A vector diagram of two tugboats applying forces to a barge in the same direction. Tugboat **A** exerts a force of 10 000 N, and tugboat **B** exerts a force of 7500 N. The resultant force vector is 17 500 N in the direction both tugs are pulling.

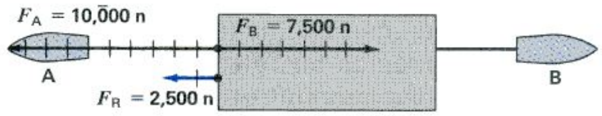


Figure 4-4. A vector diagram of two tugboats applying forces to a barge in opposite directions. Tugboat **A** exerts a force of 10 000 N. Tugboat **B** exerts a force of 7500 N. The resultant force vector is 2500 N in the direction of tugboat **A**.

The sum of two or more vectors is called the resultant.

Concurrent forces act through the same point at the same time.

Several concurrent forces can be combined into a single resultant that has the same effect.

When two or more forces act on the same point at the same time, they are called ***concurrent forces.*** *A* ***resultant force*** *is a single force that has the same effect as two or more concurrent forces.* When two forces act concurrently in the same or in opposite directions, the resultant has a magnitude equal to the algebraic sum of the forces and acts in the direction of the greater force.

When two forces act concurrently at an angle other than 0° or 180°, the resultant can be found by the parallelogram method, as in the velocity vector problems in Chapter 3. Suppose one force of 10.0 N, F_E, acts eastward upon an object at a point **O**. Another force of 15.0 N, F_S, acts southward upon the same point. Since these forces act concurrently upon point **O**, the vector diagram is constructed with the tails of both vectors at **O**. See Figure 4-5. F_E tends to move the object eastward. F_S tends to move the object southward. When the forces act simultaneously, the object tends to move along the diagonal of the parallelogram of which the two forces are sides. This is the vector F_R. The resultant force vector of two forces acting at an angle upon a given point is equal to the diagonal of a parallelogram of which the two force vectors are sides.

The graphic solution relies on a scale diagram. The trigonometric solution makes use of the facts that the opposite sides of a parallelogram are equal and that the diagonal of a parallelogram divides it into two congruent triangles. Vectors at right angles are a special case, as shown in the following example.

EXAMPLE Calculate the (a) direction and (b) magnitude of the resultant of the two forces acting at right angles on point **O** in Figure 4-5.

Given	Unknown	Basic equations
$F_E = 10.0$ N	θ	$\tan\theta = \dfrac{F_E}{F_S}$
$F_S = 15.0$ N	F_R	$\cos\theta = \dfrac{F_S}{F_R}$

Solution

(a) *Working equation:* $\theta = \arctan\left(\dfrac{F_E}{F_S}\right) = \arctan\left(\dfrac{10.0\text{ N}}{15.0\text{ N}}\right)$

$= 33.7°$ east of south

(b) *Working equation:* $F_R = \dfrac{F_S}{\cos\theta} = \dfrac{15.0\text{ N}}{0.833}$

$= 18.0$ N

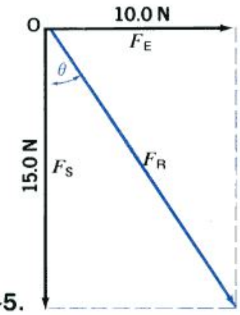


Figure 4-5.

The angle between two forces acting on the same point is often not a right angle (see Figure 4-6). The parallelogram is completed as shown. Observe that the resultant *is drawn from the point on which the two original forces are acting since the resultant will also act on this point*. The magnitude of $\boldsymbol{F_R}$ is found graphically to be 23 N. The direction is 30° south of east. (Compare these values to the corresponding ones in the example above.)

The resultant is very different if the angle between the same two forces is 140.0°. The parallelogram is constructed to scale in the same manner. The force vectors are the sides and the angle between them 140.0°. This parallelogram is shown in Figure 4-7. The diagonal must be drawn from **O,** the point at which the two forces act. The graphic solution for $\boldsymbol{F_R}$ yields 10 N 8° west of south.

The resultant of two forces acting at an acute or obtuse angle can be found trigonometrically by the laws of sines and cosines. The use of these equations is shown in the following example.

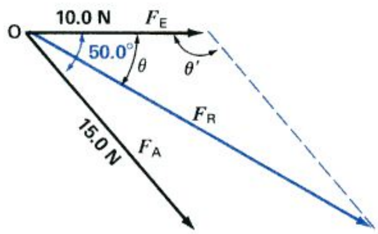


Figure 4-6. The resultant when two vectors are at acute angles.

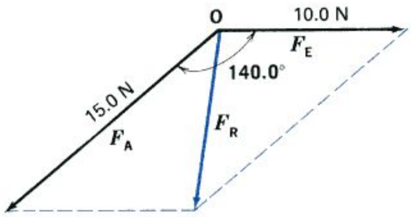


Figure 4-7. The resultant when two vectors are at obtuse angles.

EXAMPLE Calculate the (a) magnitude and (b) direction of the resultant of the two forces acting at an angle of 50.0° on point **O**, as shown in Figure 4-6.

Given	Unknown	Basic equations
$F_E = 10.0$ N	F_R	$F_R = \sqrt{F_E{}^2 + F_A{}^2 - 2F_EF_A \cos \theta'}$
$F_A = 15.0$ N	θ	$\dfrac{F_A}{\sin \theta} = \dfrac{F_R}{\sin \theta'}$
$\theta' = 130.0°$		

Solution

(a) *Working equation:*

$$\begin{aligned} F_R &= \sqrt{F_E{}^2 + F_A{}^2 - 2F_EF_A \cos \theta'} \\ &= \sqrt{(10.0\text{ N})^2 + (15.0\text{ N})^2 - 2(10.0\text{ N})(15.0\text{ N})(\cos 130.0°)} \\ &= 22.8\text{ N} \end{aligned}$$

(b) *Working equation: sin* $\theta = \dfrac{F_A \sin \theta'}{F_R}$

$$\begin{aligned} \theta &= \arcsin\left[\frac{(15.0\text{ N})(\sin 130.0°)}{(22.8\text{ N})}\right] \\ &= 30.3° \text{ south of east} \end{aligned}$$

PRACTICE PROBLEMS 1. Two forces act concurrently at right angles on point **O.** One force of 30.0 N acts south. The other force acts west with a magnitude of 40.0 N. Calculate the magnitude and direction of the resultant. *Ans.* 50.0 N 36.9° south of west

2. A force of 3.50 N acts north on point **O.** A second force of 8.75 N acts concurrently on point **O,** but at an angle of 30.0° west of north. Calculate the magnitude and direction of the resultant force.
Ans. 11.9 N 21.6° west of north

An equilibrant force is equal in magnitude to the resultant of two or more concurrent forces and acts in the opposite direction.

4.3 The Equilibrant Force

Equilibrium *is the state of a body in which there is no change in its motion.* A body in equilibrium is either at rest with respect to other bodies or moving at constant speed in a straight line. In this section we shall discuss the conditions for equilibrium of bodies at rest. The same conditions hold for the equilibrium of bodies that are in motion.

A body at rest must be in both translational and rotational equilibrium. ***Translational equilibrium*** *is the state in which there are no unbalanced (net) forces acting on a body.* The second condition of equilibrium deals with rotation. This will be discussed in Section 4.12.

When there are no unbalanced forces acting on a body, the vector sum of all the forces acting on the body is zero. For example, if a person pulls on a rope with a force of $8\bar{0}$ N and another person pulls on the same rope in the opposite direction with a force of $8\bar{0}$ N, the vector sum of the two forces is zero and the system is in equilibrium. We can also say that each force is the *equilibrant* of the other.

To find the equilibrant of two concurrent forces, we first find their resultant. Then, since the equilibrant must balance the effect of this resultant, it must have the same magnitude but act in the opposite direction. See Figure 4-8.

The equilibrant of three or more concurrent forces can be found in a similar way. First find the resultant by vector addition. Then the equilibrant is graphically drawn from the origin of the forces so that it is equal in magnitude to the resultant but extends in the opposite direction. *When two or more forces act concurrently at a point, the* ***equilibrant force*** *is that single force that if applied at the same point produces equilibrium.*

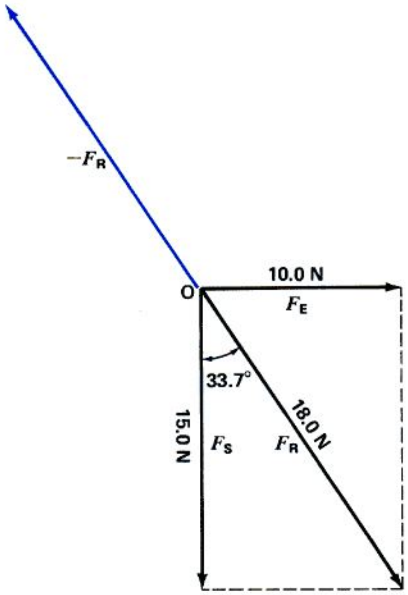


Figure 4-8. Equilibrium results when the resultant (F_R) is counterbalanced by its equilibrant ($-F_R$). Both forces must act on the same point (**O**).

QUESTIONS: GROUP A

1. What are the two ways in which objects can exert forces on one another? Give examples.
2. List four characteristics of forces.
3. What characteristic of springs allows us to use them to measure forces?
4. (a) How is the magnitude of a force shown? (b) How is the other part of this vector quantity shown?
5. (a) What are concurrent forces? (b) How are they related to the resultant force?
6. From what point is the resultant vector drawn?

GROUP B

7. (a) What is meant by *equilibrium*? (b) Is it possible for a moving object to be in equilibrium?
8. (a) If two forces acting on an object cause equilibrium, what is the sum of the two vectors? (b) What is the angle between the two vectors? (c) Is it possible for two concurrent forces, one of 5.0 N and another of 10.0 N, to cause equilibrium?
9. (a) What is an equilibrant force? (b) What is its relation to the resultant?

PROBLEMS: GROUP A

Note: Where appropriate, solve the problems both graphically and trigonometrically.

1. A classmate grabs your arm and pulls to the left with a force of 30 N. Another pulls your other arm to the right with a force of 50 N. Find the resultant force.

2. Two soccer players kick a ball at the same instant. One strikes with a force of 65 N north and the other 88 N east. Find the resultant force on the ball.
3. Find the magnitude of the resultant of the two force vectors shown in Figure 4-7 by the trigonometric method.
4. Two children pull a wagon by exerting forces of 15 N and 18 N at the same point. If the angle between them is 35.0°, what is the magnitude of the resultant force on the wagon?
5. A boy and a girl carry a 12.0-kg bucket of water by holding the ends of a rope with the bucket attached at the middle. If there is an angle of 100.0° between the two segments of the rope, what is the tension in each part?
6. Three men are pulling on ropes attached to a tree. The first man exerts a force of 6.0 N north, the second a force of 35 N east, and the third 40 N 30.0° east of south. (a) Find the resultant force on the tree by using the graphic solution. (b) What is the equilibrant force?

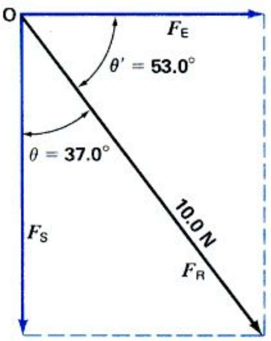


Figure 4-9. A diagram for the determination of the perpendicular components of a force.

A single force can be resolved into two or more components that have the same effect.

RESOLUTION OF FORCES

4.4 Components of Force Vectors Frequently a force acts on a body in a direction in which the body cannot move. For example, gravitational force pulls vertically downward on a wagon on an incline, but the wagon can move only along the incline. Finding the magnitude of the force that is pulling the wagon along the incline is an example of *resolution of forces*. Instead of a single force, two forces acting together can have the same effect as the original single force. One of the forces can be parallel to the surface of the incline and can pull the wagon along the incline. The other can be perpendicular to the surface of the incline. This force does not contribute to the force along the incline. These forces are at right angles to each other. As we saw in Section 2.11, two vectors that have the same effect as a single vector are called the *components* of the original vector. *This procedure of finding component forces is called* ***resolution of forces.*** Most of the examples we shall consider involve resolving a force into components that are at right angles to each other.

Before working problems with objects on an incline, study the following example, in which a force is resolved into two perpendicular components.

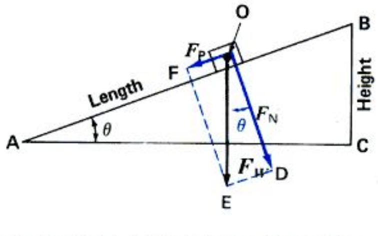


Figure 4-10. Resolution of gravitational force. One component acts parallel to the plane while the other component acts normal (perpendicular) to the plane.

4.5 Resolving Gravitational Forces An object placed on an inclined plane is attracted by the earth. The force of attraction is the weight of the object. See Figure 4-10. The plane prevents the motion of the object in the direction of F_W, the direction in which the earth's attraction acts. The vector representing the force of attraction can, however, be resolved into two components. One component acts in a direction perpendicular to the surface of the plane. In physics, the term *normal* is often used to mean perpendicular. Hence we label the normal component F_N. The other component, F_P, acts parallel to the plane. We choose these two components because they have physical significance. The vector F_N represents the force exerted by the object perpendicular to the incline or the amount of the object's weight supported by the incline. The vector F_P represents the component that tends to move the object down the incline. (The plane is assumed to be frictionless.)

Using F_W as the diagonal, we can construct the parallelogram **ODEF** and find the relative values of the sides F_P and F_N by plotting to scale. We can also express these values trigonometrically. Since right triangles **ABC** and **OED** have mutually perpendicular, or parallel, sides, the triangles are similar and $\angle \mathbf{EOD} = \theta$. Hence $\sin\theta$ can be expressed either as $\frac{\mathbf{BC}}{\mathbf{AB}}$ or $\frac{F_P}{F_W}$. This equation means that the force vector parallel to the plane, F_P, is smaller in magnitude than the weight vector, F_W, in the same ratio as the height of the plane, **BC**, is smaller than its length, **AB.**

By using $\cos\theta$, it may be similarly shown that the magnitude of the force vector perpendicular to the plane, F_N is related to the weight vector of the object in the same way that the base of the plane is related to its length.

If θ and F_W are known,

$$F_P = F_W \sin\theta$$

and

$$F_N = F_W \cos\theta$$

Making the plane steeper increases the component F_P and decreases the component F_N. This steeper inclined plane is shown in Figure 4-11. The vector F_A, which is equal and opposite to F_P, represents the applied force needed to keep the object from sliding down the plane. The steeper the plane, the greater this force becomes. It should be noted that in both Figure 4-10 and Figure 4-11, the normal force that the plane exerts on the object in reaction to force F_N is not shown, since it is not involved in the calculations for finding F_A.

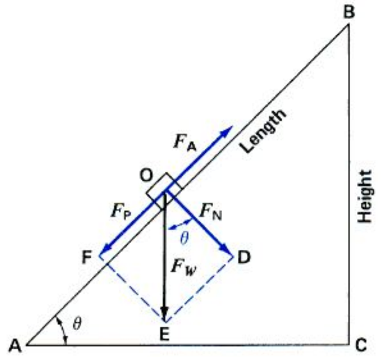


Figure 4-11. When the angle of the incline increases, the component of the weight acting parallel to the plane increases, while the component that acts normal (perpendicular) to the plane decreases.

EXAMPLE A force of 10.0 N acts on point **O** at an angle of 37.0° east of south, as shown in Figure 4-9. Find the magnitudes of the (a) eastward and (b) southward components of its force vectors.

Given	Unknown	Basic equations
$F_R = 10.0$ N	F_E	$\sin\theta = F_E/F_R$
$\theta = 37.0°$	F_S	$\cos\theta = F_S/F_R$

Solution

(a) *Working equation:*
$F_E = F_R \sin \theta$
$= (10.0 \text{ N})(\sin 37.0°)$
$= 6.02$ N due east

(b) *Working equation:*
$F_S = F_R \cos \theta$
$= (10.0 \text{ N})(\cos 37.0°)$
$= 7.99$ N due south

PRACTICE PROBLEM A lawn mower has a mass of 130 kg. A person tries to push the mower up a hill that is inclined 15° to the horizontal. How much force does the person have to exert along the handle just to keep the mower from rolling down the hill, assuming that the handle is parallel to the ground? *Ans.* 330 N

QUESTIONS: GROUP A

1. (a) A 5.0-N force and an 8.0-N force act at a point. What is the maximum resultant of these forces? (b) What is the minimum resultant? (c) Is it possible for these forces to cause equilibrium?
2. What is meant by "resolving" a force?
3. (a) Into what two components is the weight of an object on an incline resolved? (b) What is the physical significance of each?
4. (a) The handle of a lawn mower you are pushing makes an angle of 60.0° with the ground. How could you increase the horizontal forward force you are applying without increasing the total force? (b) What are some disadvantages of doing this?

GROUP B

5. What effect does making the plane in Figure 4-11 steeper have on F_N and F_P?
6. (a) What is the relationship between F_W and F_N when the angle of inclination is 0°? (b) At what angle will the components F_P and F_N have equal magnitude?
7. A skier is coming down a steep hill. (a) Which component of her weight pushes her against the hill? (b) Which component causes her to slide down the hill?
8. Which would require more force—pushing an object up to the top of the incline shown in Figure 4-11, or lifting it straight up? Why?

PROBLEMS: GROUP A

1. A child pulls a toy by exerting a force of 15 N on a string making an angle of 55° with the floor. Find the vertical and horizontal components of the force.
2. A person pushes a grocery cart by exerting a 76-N force on the handle inclined at 40.0° above the horizontal. (a) What component of the force pushes the cart against the floor? (b) What component of the force moves it forward?
3. A 20 000-N car is parked on an incline that makes an angle of 30.0° with the horizontal. If the maximum force the brakes can withstand is 12 000 N, will the car remain at rest?
4. Find the x and y components (i.e. the horizontal and vertical components) of an 88-N force making an angle of 22.0° with the x axis.

GROUP B

5. Two paramedics are carrying a person on a stretcher. One of the paramedics exerts a force of 350 N at 58° above the horizontal and the other exerts a force of 410 N at 43° above the horizontal. What is the total upward force exerted by the paramedics?
6. Ms. Jones has attached a sign that has a weight of 495 N to a wall outside her office, as shown in Figure 4-12. Determine (a) the magnitude of the tension in the chain and (b) the thrust force exerted by the rod.
7. A traffic light is supported by two wires, as shown in Figure 4-13. If the maximum tension in each wire is 750 N, what is the maximum weight of the light they can support?
8. A 2.00×10^3-kg car is to be held on a 20.0° incline by a rope in which the maximum tension is 8.00×10^3 N.
(a) Will the rope support the car?
(b) If the rope is released, how far will the car have moved down the incline by the time its speed reaches 35.0 m/s?

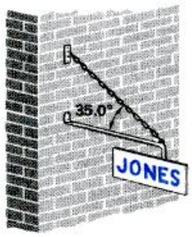


Figure 4-12.

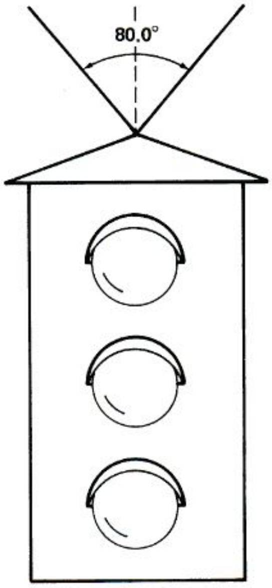


Figure 4-13.

FRICTION

4.6 The Nature of Friction In Section 4.5, we discussed the resolution of the weight of an object resting on an incline. One of the components of the weight tends to pull the object down the incline. As the angle of the incline increases, this component also increases. The slightest angle of incline will produce the component that pulls the object down the incline *if* there is no restraining force on the object. However, in performing experiments of this type, we find that the object does not begin to slide until the component parallel to the incline reaches a certain value. This means that forces must exist between the object and the incline that prevent the object from sliding. These forces are called *forces of friction*, or simply *friction*. ***Friction** is a force that resists motion. It involves objects that are in contact with each other.*

The causes of friction are sometimes complicated. Take, for example, a book lying on a table: the book's weight slightly deforms the surface of the table, along with that of the book. A "plowing" force is required to move the book over these deformations. The irregularities on both surfaces tend to interlock and offer resistance to the sliding of the book. In the process, tiny particles are torn from one surface and become imbedded in the other.

From this example one would expect that if the two surfaces are carefully polished, sliding friction between them would be lessened. Experiments have shown, however, that there is a limit to the amount by which friction may be reduced by polishing the surfaces. If they are made very smooth, the friction between them actually increases. This observation shows that some cases of sliding friction are caused by the forces of attraction between the molecules of substances.

Without friction you couldn't write your homework.

In many instances, friction is very desirable. We would be unable to walk if there were no friction between the soles of our shoes and the ground. There must be friction between the tires of an automobile and the road before the automobile can move. When we apply the brakes on the automobile, the friction between the brake linings and the brake drums, or disks, slows down the wheels. Friction between the tires and the road brings the car to a stop. In a less obvious way, friction holds screws and nails in place and it keeps dishes from sliding off a table if the table is not perfectly level. On the other hand, friction can also be a disadvantage, as it is when we try to move a heavy piece of furniture by sliding it across the floor.

4.7 Measuring Friction Friction experiments are not difficult to perform, but the results are not always easy to express as equations or laws. The following statements, therefore, should be understood as approximate descriptions only. Furthermore, they deal exclusively with solid objects. Frictional forces involving liquids and gases are beyond the scope of this book. Also, our discussion is restricted to starting and sliding friction. *Starting friction* is the maximum frictional force between stationary objects. *Sliding friction* is the frictional force between objects that are sliding with respect to one another. Static friction (which varies from 0 to the value of starting friction) and rolling friction are not considered in this text.

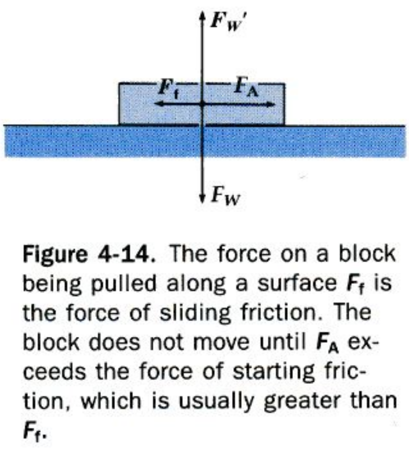


Figure 4-14. The force on a block being pulled along a surface F_f is the force of sliding friction. The block does not move until F_A exceeds the force of starting friction, which is usually greater than F_f.

1. *Friction acts parallel to the surfaces that are in contact and in the direction opposite to the motion of the object or to the net force tending to produce such motion.* Figure 4-14 illustrates this principle. The weight of the block, F_W, is balanced by the upward force of the table, F_W'. The force F_A is sliding the block along the table top. In this case, F_A is parallel to the table top. The sliding frictional force, F_f, also parallel

to the table top, resists the motion and is exerted in a direction opposite to that of F_A.

2. *Friction depends on the nature of the materials in contact and the smoothness of their surfaces.* The friction between two pieces of wood is different from the friction between wood and metal.

3. *Sliding friction is less than or equal to starting friction.* Starting friction prevents motion until the surfaces begin to slide. When the object begins to slide, less force is required to keep it sliding than was needed to start it sliding.

4. *Friction is practically independent of the area of contact.* The force needed to slide a block along a table is almost the same whether the block lies on its side or on its end. Figure 4-15 illustrates this principle. When the block is on its end, the increased pressure causes the actual area of contact to be the same as when the block is on its side.

5. *Starting or sliding friction is directly proportional to the force pressing the two surfaces together.* It does not require as much force to slide an empty chair across the floor as it does to slide the same chair when a person is sitting on it. The reason for this is that the extra force actually deforms the surfaces to some extent and thus increases the friction.

A simple way to measure starting and sliding friction is with a spring balance, as shown in Figure 4-15. Blocks of the same substance, but with different sizes and shapes, are pulled along a smooth surface. If the surfaces in contact are consistently smooth, the ratio between the force of sliding friction and the weight of the block is the same in each trial. The ratio depends only on the substances used and not on the area of contact or the weight of the block. This ratio is called the ***coefficient of sliding friction.*** It may be defined as *the ratio of the force of sliding friction to the normal* (perpendicular) *force pressing the surfaces together*. As an equation, it can be written as

$$\mu = \frac{F_f}{F_N}$$

where F_f is the force of sliding friction, μ (the Greek letter mu) is the coefficient of sliding friction, and F_N is the nor-

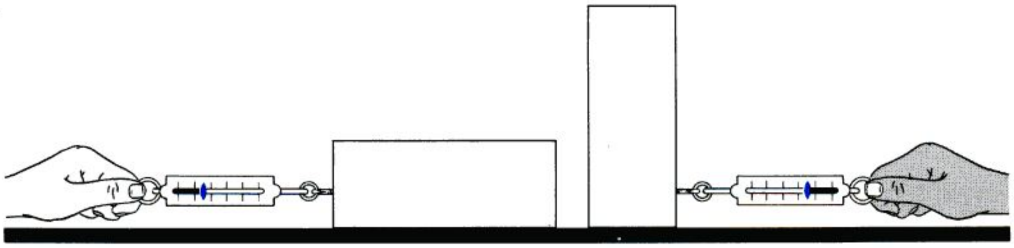


Figure 4-15. The force of friction does not vary significantly with the area of contact if the two blocks have the same weight and all surfaces are equally smooth.

Table 4-1
COEFFICIENTS OF FRICTION

Surfaces	*Starting friction*	*Sliding friction*
steel on steel	0.74	0.57
glass on glass	0.94	0.40
wood on wood	0.50	0.30
rubber tire on dry road		0.70
rubber tire on wet road		0.50
Teflon on Teflon	0.04	0.04

mal (perpendicular) force between the surfaces. (Within certain limits, this equation is an approximate summary of friction measurements.) The coefficient of starting friction is determined in a similar fashion, except that F_f is the force of starting friction. The approximate values for the coefficients of friction of various surfaces in contact with each other are given in Table 4-1.

Figure 4-16. Friction between skis and snow is appreciably reduced with the application of a wax layer on the wood or metal surface of the skis.

4.8 Changing Friction In winter, we sand icy sidewalks and streets in order to increase friction. Tire chains and snow tires are used for the same reason. In baseball, pitchers often use rosin to get more friction between their fingers and the ball. Many more examples could be given in which friction is purposely increased by changing the nature of the surfaces that are in contact.

The most common method of reducing sliding friction is by lubrication. The skier pictured in Figure 4-16 applied a layer of wax to the skis to reduce friction. A thin film of oil between rubbing surfaces reduces friction. The lesser friction between a liquid and a solid has replaced the greater friction between two solids. Alloys have also been developed that are in effect self-lubricating. For example, when steel slides over an alloy of lead and antimony, the coefficient of friction is less than when steel slides over steel. Bearings lined with such an alloy reduce friction. From Table 4-1 it is also obvious that if a bearing is coated with a plastic such as Teflon, there is very little friction. Such bearings are used in electric motors where the use of a liquid lubricant is undesirable.

Friction may also be greatly reduced through the use of ball bearings or roller bearings. Sliding friction is changed to rolling friction, which has a much lower coefficient. Using steel cylinders to roll a heavy box along the floor is another example of changing sliding to rolling friction.

The squeaking wheel gets the grease.

4.9 Solving Friction Problems The force required to slide an object along a level surface can be computed easily from the weight of the object and the coefficient of sliding friction between the two surfaces. However, when the force applied to the object is not applied in the direction of the motion, it is necessary to resolve forces in the calculation. This resolution of forces is illustrated in the following example.

In the case of an object resting on an incline, the forces of starting and sliding friction will determine whether the object remains at rest, slides down the incline with constant speed, or accelerates as it descends. How the angle of the incline and coefficient of sliding friction are used in such a problem is illustrated in another example.

EXAMPLE A box weighing $45\bar{0}$ N is pulled along a level floor at constant speed by a rope that makes an angle of 30.0° with the floor, as shown in Figure 4-17. If the force on the rope is $26\bar{0}$ N, (a) what is the horizontal component (F_h) of this force? (b) What is the normal force (F_N)? (c) What is the coefficient of sliding friction (μ)?

Given	Unknown	Basic equations
$F_W = 45\bar{0}$ N	F_h	$\cos\theta = \dfrac{F_h}{F_A}$
$\theta = 30.0°$	F_N	$F_N = F_W - F_A \sin\theta$
$F_A = 26\bar{0}$ N	μ	$\mu = \dfrac{F_f}{F_N}$
$F_f = F_h$		

Solution

(a) *Working equation:* $F_h = F_A \cos\theta$
$= (26\bar{0}\text{ N})(0.866)$
$= 225$ N

(b) The normal force (F_N) is the difference between the downward force of the block's weight (F_W) and the vertical component of the force of the rope (F_v):

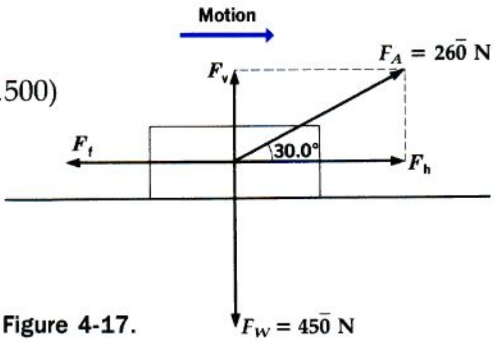


Figure 4-17.

Working equation: $F_N = F_W - F_A \sin\theta$
$= 45\bar{0}\text{ N} - (26\bar{0}\text{ N})(0.500)$
$= 32\bar{0}$ N

(c) *Working equation:* $\mu = \dfrac{F_f}{F_N} = \dfrac{F_h}{F_N}$
$= \dfrac{225\text{ N}}{32\bar{0}\text{ N}}$
$= 0.703$

EXAMPLE A wooden block weighing $13\bar{0}$ N rests on an inclined plane, as shown in Figure 4-18. The coefficient of sliding friction between the block and the plane is 0.620. Find the angle of the inclined plane at which the block will slide down the plane at constant speed once it has started moving.

Given	Unknown	Basic equations
$F_W = 13\bar{0}$ N	θ	$\tan\theta = \dfrac{F_P}{F_N}$
$\mu = 0.620$		$\mu = \dfrac{F_f}{F_N}$

Solution

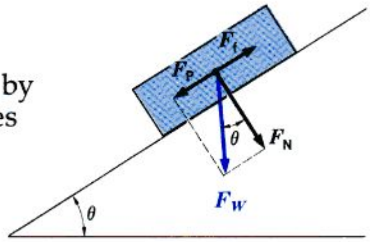


Figure 4-18.

The force along the incline (F_P) is opposed by the force of friction (F_f). When the block slides down at constant speed, $F_P = F_f$.

Substituting this value into the first Basic equation: $\tan\theta = \dfrac{F_f}{F_N}$

Notice that the term on the right is also the definition for the coefficient of friction. From this we get the

Working equation: $\tan\theta = \mu$

$$\theta = \arctan \mu = \arctan (0.620)$$
$$= 31.8°$$

In other words, the block will slide at constant speed when the angle of the incline is 31.8°, no matter what the block weighs. (The weight of the block cancels out.) If the angle is greater than 31.8°, the block will accelerate as it slides down the plane. The angle at which the block slides depends only on the coefficient of friction.

PRACTICE PROBLEMS **1.** A box with a mass of 175 kg is pulled along a level floor with constant velocity. If the coefficient of friction between the box and the floor is 0.34, what horizontal force is exerted in pulling the box? *Ans.* 583 N

2. A crate is pulled with constant velocity up an inclined floor that makes an angle of 12° with the horizontal. The crate weighs 950 N and the pulling force parallel to the floor is 460 N. Find the coefficient of friction between the crate and the floor. *Ans.* 0.28

QUESTIONS: GROUP A

1. What is friction?
2. State two theories that scientists use to explain the causes of friction.
3. Make a list of the places you encounter frictional effects as you get ready to go to school. Indicate if the effects are useful.
4. When an object slides across a surface, what is the direction of the frictional force on the object?
5. What are the factors affecting solid, sliding friction?
6. Why is the amount of friction seemingly independent of the surface areas in contact?

GROUP B

7. (a) What happens to the coefficient of friction between tires and the road on a rainy day? (b) How should a driver compensate for this effect when approaching a stop sign?
8. Compare friction between solids sliding and solids rolling across a surface.
9. How does static friction change as you push downward on a stationary object?
10. Indicate if the force of friction will increase or decrease when (a) sand is thrown on icy streets, (b) bearings in machinery are lubricated, and (c) car tires are worn down.

PROBLEMS: GROUP A

1. A horizontal force of 400.0 N is required to pull a 1760-N trunk across the floor at constant speed. Find the coefficient of sliding friction.
2. How much force must be applied to push a 1.35-kg book across the desk at constant speed if the coefficient of sliding friction is 0.30?
3. A force of 105 N is applied horizontally to a 20.0-kg box to move it across a horizontal floor. If the box has an acceleration of 3.00 m/s^2, find the coefficient of friction.
4. A 1500.0-N force is exerted on a 200.0-kg crate to move it across the floor. If the coefficient of friction is 0.250, what is the crate's acceleration?
5. A 100.0-kg commuter is standing on a train accelerating at 3.70 m/s^2. What coefficient of friction must exist between the commuter's feet and the floor to avoid sliding?

GROUP B

6. A 146-N force is used to pull a 350-N wood block at constant speed by a rope making an angle of 50.0° with the floor. Find the coefficient of sliding friction.
7. A 75.0-kg baby carriage is pushed along a level sidewalk by exerting a force of 50.0 N on the handle, which makes an angle of 60.0° with the horizontal. What is the coefficient of friction between the carriage and the sidewalk?
8. A 3.00-kg wood box slides from rest down a 35.0° inclined plane. How long does it take the box to reach the bottom of the 4.75-m wood incline? (See Table 4-1 for the coefficient of friction.)
9. A 65.0-kg crate is to be accelerated at 7.00 m/s^2 up an incline making a 25.0° angle with the horizontal. If the coefficient of sliding friction between the crate and the incline is 0.200, how much force is required?
10. A 60.0-kg crate is attached to a weight by a cord that passes over a frictionless pulley, as shown in Figure 4-19. (a) If the coefficient of friction is 0.500, what weight will keep the crate moving up the 40.0° incline at a constant speed? (b) If the cord is cut when the crate is at rest at the top of the incline, how far would the crate have slid by the time its speed reached 7.50 m/s?
11. If the coefficient of friction between a set of waxed skis and the snow is 0.10, at what angle will a 90.0-kg skier move at a constant speed down the slope?

PARALLEL FORCES

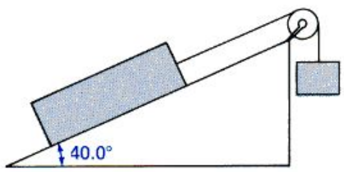


Figure 4-19.

4.10 Center of Gravity Thus far in our study of forces, we have been treating all the forces acting on a body as if they were acting at a single point. However, there can be many forces, each acting at a different point on the object. For example, Figure 4-20 represents a stone lying on the ground. Since every part of the stone has mass, every part is attracted to the center of the earth. Because of the large size of the earth, all the downward forces exerted on the stone are virtually parallel. The weight of the stone can be thought of as a force vector that is the vector sum, or resultant, of all these parallel force vectors. ***Parallel forces***

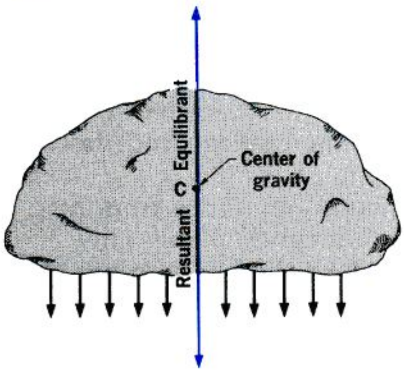


Figure 4-20. The stone's center of gravity is the point where all the weight seems to be concentrated.

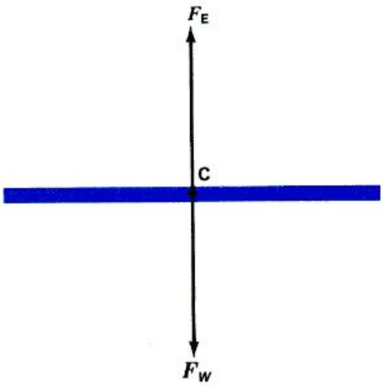


Figure 4-21. The weight of the bar, F_W, is apparently concentrated at the center of gravity, C, and can be balanced by an equal and opposite force, F_E, applied at C.

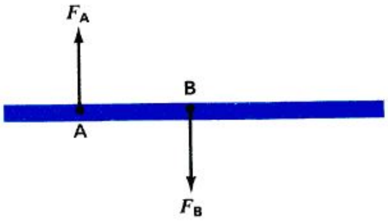


Figure 4-22. Two parallel forces, F_A and F_W, act at different points. A tendency to rotate results.

act in the same or in opposite directions at different points on an object. The resultant of parallel forces has a magnitude equal to the algebraic sum of all the forces. The resultant acts in the direction of this net force.

But where in the stone is this resultant force acting? Experiments show that if the proper point of application is chosen, the stone can be lifted without producing rotation. As shown in Figure 4-20, the equilibrant lifting force vector is then in line with the resultant (weight) vector of the stone. In other words, the stone acts as if all its weight were located at one point, which is called the *center of gravity. The **center of gravity** of any object is that point at which all of its weight can be considered to be concentrated.*

In Figure 4-21, the center of gravity of the bar is at **C.** In a bar of uniform construction, **C** is at the geometric center. But if the density or shape of the bar is not uniform, **C** is not at the geometric center.

Since the weight of the bar, F_W, can be considered to be acting at **C,** the bar can be suspended without changing its rotation by an equilibrant force, F_E, applied at **C.** Since F_W and F_E are equal but opposite vectors, they counterbalance each other. In this condition, the bar is in both *translational and rotational equilibrium.* This means that the bar is not accelerating and its rotation (if any) is constant.

4.11 Torques The two forces represented in Figure 4-22 by the vectors F_A and F_B are parallel. They do not act on the same point as did the concurrent forces we studied earlier in this chapter. To measure the rotating effect, or *torque,* of such parallel forces in a given plane, it is first necessary to choose a stationary reference point for the measurements. We shall refer to this stationary reference point as the *pivot point.*

Sometimes, as in the case of a seesaw, there is a "natural" point about which the rotating effects can be measured. However, such a pivot point is "natural" only when the seesaw is in motion. When it is motionless there is no "natural" pivot point. Any point on the seesaw, or even beyond it, can be chosen.

Once a suitable pivot point is chosen, a perpendicular line is drawn on the vector diagram from it to each of the lines along which force vectors act on the object. Each such line is called a *torque arm.* In some cases, the force vectors must be extended in order to meet the perpendicular. ***Torque,*** *T, is the product of a force and the length of its torque arm.* The unit of torque is the meter-newton.

To illustrate the concept of torque, consider the bar in Figure 4-22 with the application of an additional force, F_C, as shown in Figure 4-23. Choosing **A** as the pivot point, **BA** is the torque arm of F_B and **CA** is the torque arm of the additional upward force, F_C. The clockwise torque around **A** is the product of F_B and **BA.** The counterclockwise torque is the product of F_C and **CA.** Since F_A has a torque arm of zero, it produces no torque and does not enter into the calculations.

To identify a torque as clockwise or counterclockwise, imagine that the bar is free to rotate around a stationary pivot point. Further imagine that the force producing the torque is the only force acting on the bar. The direction in which the bar would rotate is the direction of the torque.

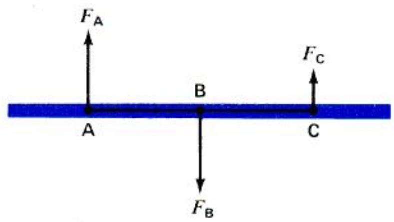


Figure 4-23. Rotational equilibrium results when the sum of the clockwise torques is equal to the sum of the counterclockwise torques. Any point may be chosen as the pivot point in making the computation provided the vector sum of the forces is zero.

4.12 Rotational Equilibrium In Section 4.3 we discussed the conditions necessary for the translational equilibrium of an object. These conditions, however, do not prevent the *rotary motion* of an object that is subjected to torques. To prevent rotation in a given plane a second condition of equilibrium must be met. ***Rotational equilibrium*** *in a given plane is the state in which the sum of all the clockwise torques equals the sum of all the counterclockwise torques about any pivot point.*

Both conditions of equilibrium are illustrated in Figure 4.24. The sum of the force vectors is zero ($3\bar{0}$ N upward

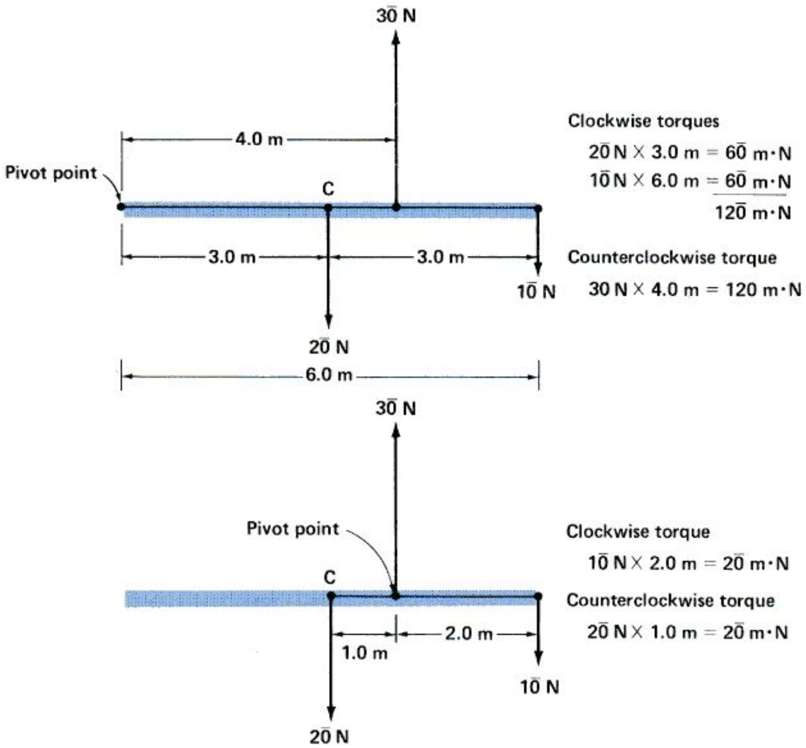


Figure 4-24. The calculation of torques can be simplified by setting one of the torque arms equal to zero. In the upper diagram, the left end of the bar is used as the pivot point. In the lower diagram, the point of application of the upward force is used, thereby reducing the number of torques.

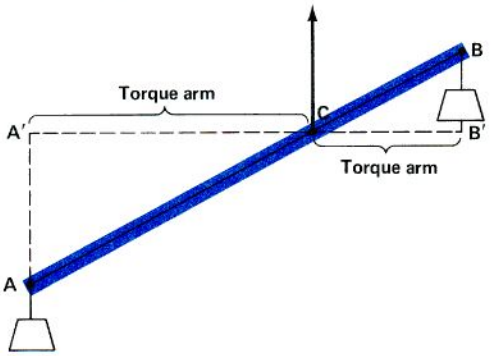


Figure 4-25. The torque arm is the perpendicular distance from the pivot point to the line indicating the direction of the applied force.

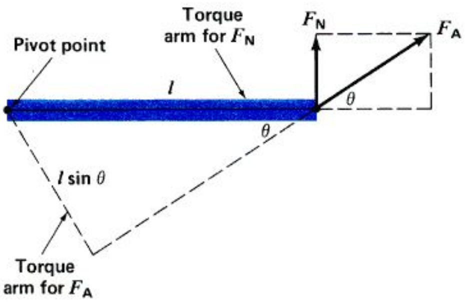


Figure 4-26. The resolution of forces is used to find the torque produced by a force acting on the bar at an angle other than perpendicular.

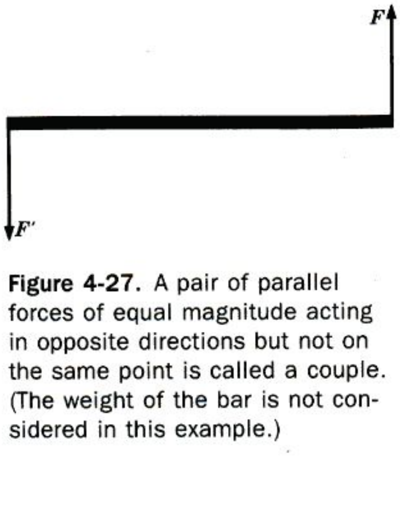


Figure 4-27. A pair of parallel forces of equal magnitude acting in opposite directions but not on the same point is called a couple. (The weight of the bar is not considered in this example.)

against $2\overline{0}$ N + $1\overline{0}$ N downward). The sum of the clockwise torques is equal to the sum of the counterclockwise torques. Two methods of computing the torques are shown. In the upper drawing of Figure 4-24, the left end of the bar is chosen as a pivot point. In the lower drawing, the point of application of the upward force is used as the pivot point. This simplifies the calculation.

When forces are applied to a bar at an angle other than perpendicular, the distances along the bar measured from the points of application of the forces cannot be used to measure the torque arms. *The torque arms must always be measured perpendicular to the directions of the forces.* Figure 4-25 shows such a situation. A meter stick is under the influence of three parallel forces. To find the torque arms, it is necessary to draw a horizontal line through the pivot point. Since the weights hang vertically, a horizontal line is perpendicular to the force vectors of the weights. The problem is simplified by placing the pivot point at the center of gravity. If the bar is not homogeneous, this center of gravity may not be located at the geometric center of the bar. (Experimentally, the center of gravity can be approximately located by finding the point where the bar balances.) Placing the pivot point at the center of gravity eliminates two torques in the equation. It eliminates the torque produced by the weight of the bar located at some distance from the geometric center of the bar. And it also eliminates the torque produced by the force acting upward at **C**. The required torque arms, **CA′** and **CB′**, are then found by multiplying the distances **CA** and **CB** by the cosine of the angle **ACA′** or **BCB′**.

In Figure 4-26, F_A is applied to the right end of the bar at an angle other than perpendicular. In order to find the counterclockwise torque produced by F_A, we choose the left end of the bar as the pivot point. Then we find the vertical component, F_N, of F_A. By trigonometry,

$$F_N = F_A \sin \theta$$

Since F_N is perpendicular to the bar, the required torque is

$$T = F_N l$$

Substituting,

$$T = F_A l \sin \theta$$

Hence, $l \sin \theta$ is the torque arm of F_A. The result is further verified in Figure 4-26, where $l \sin \theta$ is the length of the perpendicular from the pivot point to the extended line of direction of F_A. This is in accord with the definition of torque arm in Section 4.11.

4.13 Coupled Forces The conditions of equilibrium hold true no matter how many forces are involved. An interesting example is one in which *two forces of equal magnitude act in opposite directions in the same plane, but not on the same point.* Such a pair of forces is called a ***couple.*** A diagram of a couple is shown in Figure 4-27. The torque is equal to the product of one of the forces and the perpendicular distance between them. (This can be proved by computing the sum of the torques produced by the action of the separate forces about any desired pivot point.) A good example of a couple is the pair of forces acting on the opposite poles of a compass needle when the needle is not pointing north and south.

A couple cannot be balanced by a single force since this single force would be unbalanced and would produce linear motion where it was applied. The only way to balance a couple is with another couple; the torques of the two couples must have equal magnitudes but opposite directions.

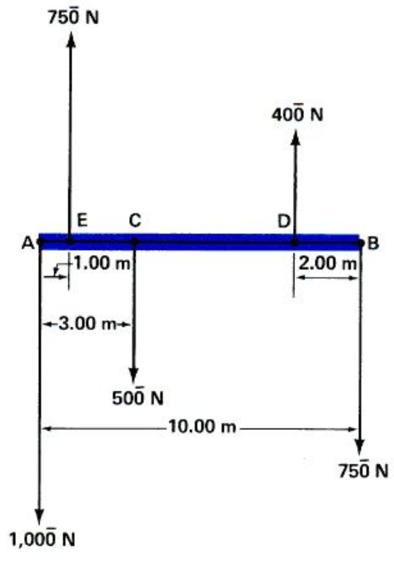


Figure 4-28.

EXAMPLE A horizontal rod, **AB,** is 10.00 m long. It weighs 50$\bar{0}$ N and its center of gravity, **C,** is 3.00 m from **A.** At **A** a force of 100$\bar{0}$ N acts downward. At **B** a force of 75$\bar{0}$ N acts downward. At **D,** 2.00 m from **B,** a force of 40$\bar{0}$ N acts upward. At **E,** 1.00 m from **A,** a force of 75$\bar{0}$ N acts upward. (a) What is the magnitude and direction of the force that must be used to produce equilibrium? (b) Where must it be applied? (See Figure 4-28.)

Solution

(a) Consider the known upward force vectors as being positive and the known downward force vectors as being negative. The algebraic sum of these force vectors will then be the resultant force vector. This resultant force vector must be counterbalanced by a force vector in the opposite direction in order to establish translational equilibrium.

$$75\bar{0}\text{ N} + 40\bar{0}\text{ N} - 100\bar{0}\text{ N} - 50\bar{0}\text{ N} - 75\bar{0}\text{ N} = -110\bar{0}\text{ N}$$

Therefore, 110$\bar{0}$ N must be applied upward to establish translational equilibrium.

(b) Use **A** as the pivot point, and let x be the distance from **A** to the point where the 110$\bar{0}$-N force must be applied to prevent rotary motion.

$$\begin{aligned}\text{clockwise torque} &= (50\bar{0}\text{ N})(3.00\text{ m}) + (75\bar{0}\text{ N})(10.00\text{ m})\\ \text{counterclockwise torque} &= (75\bar{0}\text{ N})(1.00\text{ m}) + (40\bar{0}\text{ N})(8.00\text{ m}) + (110\bar{0}\text{ N})x\\ 90\bar{0}0\text{ m}\cdot\text{N} &= 3950\text{ m}\cdot\text{N} + (110\bar{0}\text{ N})x\\ x &= 4.59\text{ m}\end{aligned}$$

4.59 m is the distance from **A** to the point where the 110$\bar{0}$-N upward force must be applied.

PRACTICE PROBLEMS **1.** A nonuniform bar is 3.8 m long and has a weight of 560 N. The bar is balanced in a horizontal position when it is supported at its geometric center and a 340-N weight is hung 0.70 m from the bar's light end. Find the bar's center of gravity.
Ans. 1.2 m from heavy end

2. A large wooden beam weighs 820 N and is 3.2 m long. The beam's center of gravity is 1.4 m from one end. Two workers begin carrying the beam away. If they lift the beam at its ends, what part of its weight does each worker lift? *Ans.* 460 N and 360 N

QUESTIONS: GROUP A

1. (a) What is meant by the term *center of gravity?* (b) Where is the center of gravity of a meterstick? A bowling ball? An ice cube? A doughnut? A banana?
2. (a) What are the conditions for equilibrium? (b) Explain how they apply to children attempting to balance a seesaw.
3. How is torque calculated?
4. Why is it easier to loosen the lid from the top of a can of paint with a long-handled screwdriver than with a short-handled screwdriver?
5. How would the force needed to open a door change if you put the handle in the middle of the door?

GROUP B

6. How does an orthodontist use torque in realigning teeth?
7. What factor determines the location of the pivot point in a torque problem?
8. How can an object on which a couple is acting be placed in equilibrium?
9. What must be true for a moving object to be in equilibrium?
10. A twirler throws a baton straight up into the air. (a) Describe the motion of the ends of the baton. (b) Describe the motion of the center of gravity of the baton.

PROBLEMS: GROUP A

Note: For each problem, draw and label an appropriate force diagram. Unless otherwise noted, the center of gravity is at the geometric center of the object.

1. A 400.0-N child and a 300.0-N child sit on either end of a 2.00-m-long seesaw. Where along the seesaw should the pivot support be placed to ensure rotational equilibrium?
2. Based on the information in Problem 1 and its solution, suppose a 225-N child sits 0.200 m from the 400.0-N child. Where must a 325-N child sit to maintain rotational equilibrium?
3. A uniform meterstick, supported at the 30.0-cm mark, is balanced when a 0.50-N weight is hung at the 0.0-cm mark. What is the weight of the meterstick?
4. A 650-N boy and a 490-N girl sit on a 150-N porch swing that is 1.70 m long. If the swing is supported by a chain at each end, what is the tension in each chain when the boy sits 0.750 m from one end and the girl 0.500 m from the other?
5. A uniform bridge, 20.0 m long and weighing 4.00×10^5 N, is supported by two pillars located 3.00 m from each end. If a 1.96×10^4-N car is parked 8.00 m from one end of the bridge, how much force does each pillar exert?

GROUP B

6. A 30.0-N fishing pole is 2.00 m long and has its center of gravity 0.350 m from the heavy end. A fisherman holds the end of the pole in his left hand as he lifts a 100.0-N fish. If his right hand is 0.800 m from the heavy end, how much force must he exert with his right hand to maintain equilibrium?
7. A uniform 2.50-N meterstick is hung from the ceiling by a single rope. A 500.0-g mass is hung at the 25.0-cm mark and a 650.0-g mass at the 70.0-cm mark. (a) What is the tension in the rope? (b) Where is the rope attached to the meterstick?
8. An 850-N painter stands 1.20 m from one end of a 3.00-m scaffold supported at each end by a stepladder. The scaffold weighs 250 N and there is a 40.0-N can of paint 0.50 m from the end opposite the painter. How much force is exerted by each stepladder?
9. A 10.0-N meterstick is suspended by two spring scales, one at the 8.00-cm mark and the other at the 90.0-cm mark. If a weight of 5.00 N is hung at the 20.0-cm mark and a weight of 17.0 N is hung at the 55.0-cm mark, what will be the reading on each scale?

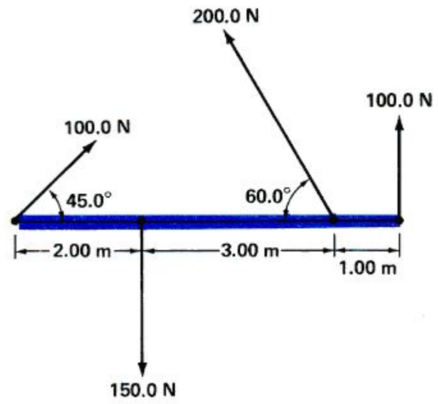


Figure 4-29.

10. (a) Find the torques exerted on the rod in Figure 4-29. (b) Find the magnitude and direction of the additional force that must be exerted at the right end, perpendicular to the rod, to maintain rotational equilibrium.

PHYSICS ACTIVITY

(a) Find the center of gravity of a broom by balancing it lengthwise on your hand. Is the center of gravity closer to the bristles or to the end of the handle?
(b) Balance the broom vertically on the palm of your hand, first with the top of the handle and then with the bristles. Which way is easier? Explain.

SUMMARY

Forces are vector quantities. A net force will change the state of motion of an object. Forces can be exerted over long distances. Every force is accompanied by an opposite force.

A resultant force is a single force that produces the same effect as several forces acting along lines that pass through the same point. The equilibrant force is the single force that produces equilibrium when applied at a point at which two or more concurrent forces are acting. A single force may be resolved into two components that usually act at right angles to each other. Resultant and component forces are found by the parallelogram method.

Friction is a force that resists the motion of objects that are in contact with each other. It acts parallel to the surfaces that are in contact, depends on the nature and smoothness of the surfaces, is virtually independent of position, and is directly proportional to the force pressing the surfaces together. Starting friction is usually greater than sliding friction. The coefficient of friction is the ratio of the force of friction to the perpendicular force pressing the surfaces together. The resolution of forces is used in solving friction problems.

Parallel forces act in the same or in opposite directions. The center of gravity of an object is that point at which all of the object's weight can be considered to be concentrated. A stationary object is in translational and rotational equilibrium. The torque produced by a force is the product of the force and the length of the torque arm on which it acts. To produce equilibrium in parallel forces, the sums of the forces in opposite directions must be equal and the sum of all the clockwise torques must equal the sum of all the counterclockwise torques about a pivot point. Two forces of equal magnitude that act in opposite directions but not along the same line are called a couple.

VOCABULARY

center of gravity
coefficient of sliding friction
concurrent forces
couple
equilibrant force
equilibrium
friction
parallel forces
resolution of forces
resultant force
rotational equilibrium
torque
torque arm
translational equilibrium