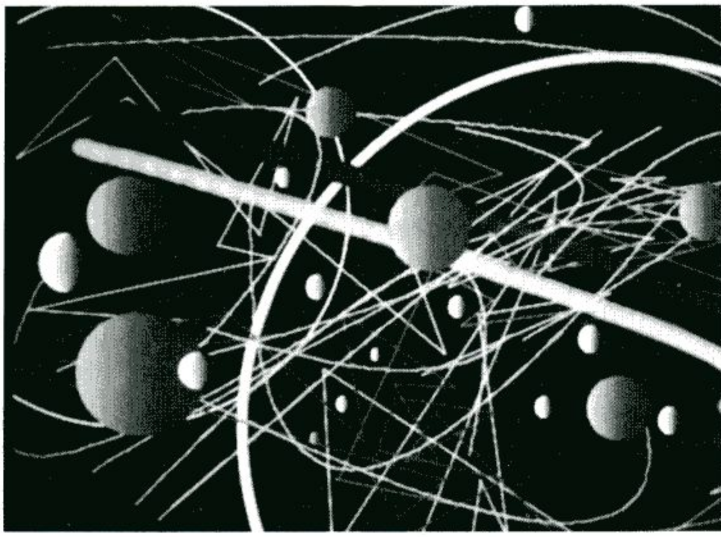
6

Conservation of Energy and Momentum

energy (EN-er-jee) n.: the capacity to do work.



WORK, MACHINES, AND POWER

6.1 Definition of Work The word *work* has a specific meaning in physics. ***Work** is done when a force is exerted on an object causing the object to move in the direction of a component of the applied force.* When you hold a heavy load on your shoulder, as long as you do not move you are not doing any work on the load. You are exerting an upward *force* that counteracts the downward force of gravity on the load. You do *work* when you raise the load to your shoulder, when you carry it up a flight of stairs, or when you pull it across the floor. In these cases, you exert a force that has a component in the direction in which the object moves.

Two factors must be considered in measuring work: the displacement of the object and the magnitude of the force in the direction of displacement. *The amount of work, W, equals the product of a displacement, Δd, and the force, F, in the direction of the displacement.*

$$W = F \Delta d$$

When the force is measured in newtons and the distance through which it acts is measured in meters, the work is expressed in *joules* (J). *A force of one newton acting through a distance of one meter does **one joule** of work.* This unit of work is named for the English physicist James Prescott Joule. Note that *a joule is a newton-meter.*

For example, let us compute the work required to lift a

OBJECTIVES

- Define and calculate work and power.
- Define and calculate kinetic energy and potential energy.
- Define and calculate impulse and momentum.
- Apply the laws of conservation of energy and momentum to solve motion problems.
- Differentiate between elastic and inelastic collisions.

1.0-kg mass to a height of 5.0 m. From the relationship between mass and weight discussed in previous chapters, we know that a force of about 9.8 N must be exerted to lift a mass of 1.0 kg at sea level. Thus, the work is

$$\begin{aligned} W &= F\Delta d \\ &= (9.8 \text{ N})(5.0 \text{ m}) \\ &= 49 \text{ J} \end{aligned}$$

If we slide the 1.0-kg mass at constant velocity along a horizontal surface having a coefficient of sliding friction 0.30 for 5.0 m, the work required is

The relationship $F_f = \mu F_N$ is explained in Section 4.7.

$$\begin{aligned} W &= F_f\Delta d \\ &= \mu F_N \Delta d \\ &= (0.30)(9.8 \text{ N})(5.0 \text{ m}) \\ &= 15 \text{ J} \end{aligned}$$

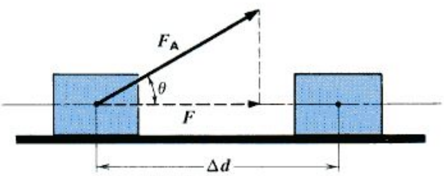


Figure 6-1. Definition of work. The work done by the applied force, F_A, is equal to the product of F, the component of F_A in the direction of the displacement, and Δd, the distance through which the mass moves.

In both instances, the force is applied in the direction in which the object moves.

Suppose, however, that the force is applied to the object in a direction other than that in which it moves. In that case, only the component of the applied force that acts in the direction the object moves is used to compute the work done on the object. Thus in Figure 6-1, the relationship between the applied force, F_A, and the component in the direction of motion, F, is

$$F = F_A \cos \theta$$

When the force is applied at an angle, θ, the normal force, F_N, is equal to the weight of the object, F_W, minus the vertical component of the applied force, or

$$F_N = F_W - F_A \sin \theta$$

This equation is used to calculate F_A when it is not given in a problem, as shown in the following example. Then

$$W = F_A \Delta d \cos \theta$$

EXAMPLE A 95.0-kg crate is pulled for 12.0 m on a horizontal surface at a constant velocity. The coefficient of friction between the crate and the ground is 0.260. Calculate the work done when the force is applied at an angle of 20.0°.

Given	Unknown	Basic equations
$m = 95.0$ kg	W	$W = \mu F_N \Delta d$
$\Delta d = 12.0$ m		$F_f = \mu F_N$
$\mu = 0.260$		$F_N = F_W - F_A \sin\theta$
$\theta = 20.0°$		$W = F_A \Delta d \cos\theta$

Solution

Step 1: $\mu = F_f/F_N = F_A \cos\theta/(mg - F_A \sin\theta)$

$$F_A = \mu mg/(\mu \sin\theta + \cos\theta)$$
$$= \frac{(0.260)(95.0\ \text{kg})(9.80\ \text{N/kg})}{(0.260)(0.342) + (0.940)} = 235\ \text{N}$$

Step 2: $W = F_A\, \Delta d \cos\theta$

$$= (235\ \text{N})(12.0\ \text{m})(0.940)$$
$$= 2.65 \times 10^3\ \text{J}$$

PRACTICE PROBLEMS **1.** A crate weighing 850 N is pushed up an inclined plane a distance of 10.0 m. The plane makes an angle of 15° with the horizontal and the crate moves with constant velocity. The coefficient of friction between the crate and the plane is 0.24. Calculate the work that is done in pushing the crate. *Ans.* 4.2×10^3 J

2. A girl pulls a wagon with constant velocity along a level path for a distance of 45 m. The handle of the wagon makes an angle of 20.0° with the horizontal, and she exerts a force of 85 N on the handle. Find the amount of work the girl does in pulling the wagon. *Ans.* 3.6×10^3 J

6.2 Work Done by Varying Forces In the examples of work in Section 6.1, the forces involved did not vary. In many problems involving work, however, the forces may vary in direction, in magnitude, or in both during the time that they are acting on an object. For example, when a force is used to stretch a spring, the magnitude of the force increases as the spring gets longer.

An easy way to determine the amount of work done by a varying force is to use a graph. In Figure 6-2, the area under the curved line represents the work done by a force that varies in magnitude. The horizontal axis is the distance, Δd, in meters and the vertical axis is the force that acts in the direction of motion, F, in newtons. The amount of work required to provide the displacement indicated by the curve up to point **A,** for example, is equal to the area bounded at the top by the curve, at the right by a vertical line from **A** to the horizontal axis, and at the left and bottom by the two coordinate axes.

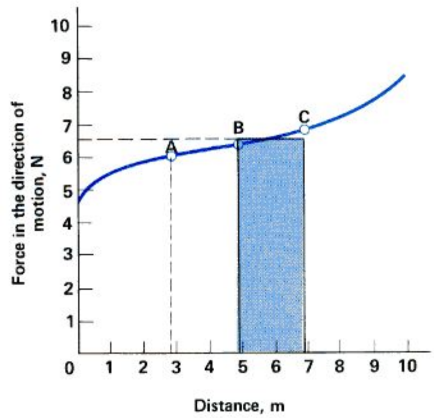


Figure 6-2. Work done by a variable force. The area under the curve represents the total work.

To calculate the area of a geometric figure that is bounded by one or more curved lines requires a branch of mathematics called calculus. A good approximation is obtained, however, by the use of suitable rectangles. For example, the work indicated by the curve between points **B** and **C** is approximately equal to the area of the blue rectangle in the figure. Thus, the work is approximately equal to the product of 2.0 m and 6.5 N, or 13 J.

The total amount of work that is represented by the area under the curve can be found by adding the areas of many rectangles formed in the same way as the one in the figure. If the rectangles are made very narrow, the answer will be more accurate. An interesting way to measure the area under a curve is to cut it out of a piece of paper, weigh it, and compare it with the weight of a known area.

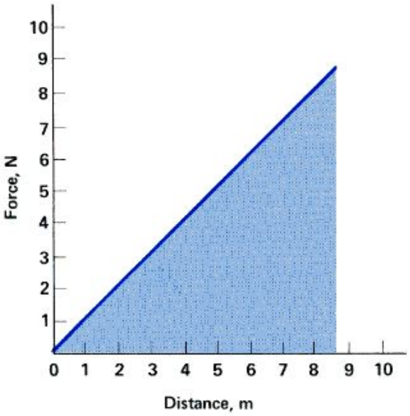


Figure 6-3. Work done by a constantly increasing force. The total work is represented by the blue triangle.

The problem of finding the total work done by a varying force is somewhat simpler in the case of a stretching spring, as long as the force is constantly applied in the direction of the stretching. The force required to stretch a spring depends on the stiffness of the spring, but the force is directly proportional to the amount of stretching. (The limits within which this relationship is true will be discussed in Chapter 7.) Consequently, a graph of the work done in stretching a spring is shown in Figure 6-3. No approximations are required in computing the total work because the area of the triangle under the curve is exactly equal to one-half the product of its base and height.

6.3 Work in Rotary Motion

To compute the work done in rotary motion, we make use of the principles of radian measure (Section 5.6). In Figure 6-4 the displacement of the rim of the wheel is designated by the arc Δd. If the angle $\boldsymbol{\Delta\theta}$ is expressed in radians, then $\Delta d = r\,\Delta\theta$. Substituting this expression for Δd in the work equation, we get

$$W = Fr\Delta\theta$$

Furthermore, in Section 4.12 we saw that a torque, T, is equal to the product of a force and the length of its torque arm. In Figure 6-4, the torque arm is the radius of the circle, so $T = Fr$. The work equation can now be written

$$W = T\Delta\theta$$

which means that the work done in rotary motion can be computed by finding the product of the torque producing the motion and the angular displacement in radians.

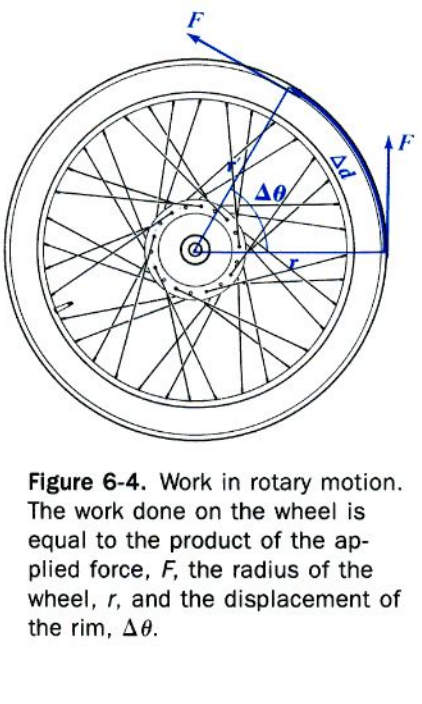


Figure 6-4. Work in rotary motion. The work done on the wheel is equal to the product of the applied force, F, the radius of the wheel, r, and the displacement of the rim, $\Delta\theta$.

For example, if the radius of the wheel in Figure 6-4 is

2.0 m and a force of 12 N is applied tangentially to it, the work done in a single revolution is

$$\begin{aligned} W &= T\Delta\theta \\ &= Fr\Delta\theta \\ &= 2\pi Fr \\ &= 2(3.14)(12\ \text{N})(2.0\ \text{m}) \\ &= 150\ \text{J} \end{aligned}$$

6.4 Machines Six types of simple machines are shown in Figure 6.5. A *machine* can be used to multiply force. Other machines are either modifications of these simple machines or combinations of two or more of them. The six machines shown are actually variations of two basic types: the pulley and the wheel and axle are forms of the lever, and the wedge and screw are modified inclined planes.

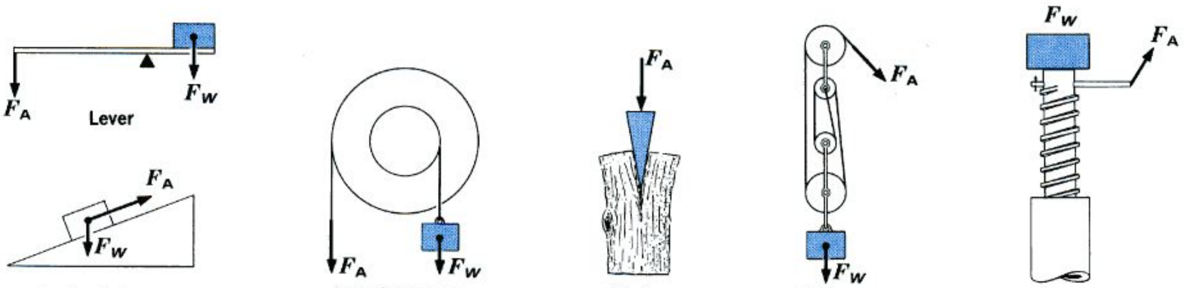


Figure 6-5. Simple machines. Each machine multiplies force at the expense of distance.

Although a machine can be used to multiply force, it cannot multiply work. The work output of a machine cannot exceed the work input. In a frictionless machine, work output and work input would be exactly equal. In a machine that multiplies force, this equality means that the distance over which the input force moves is always greater than the distance over which the load moves.

The ratio of the output force to the input force in a machine is called the mechanical advantage.

The ratio of the useful work output of a machine to total work input is called the ***efficiency.***

$$\textbf{Efficiency} = \frac{W_{\text{Output}}}{W_{\text{Input}}}$$

The efficiency of all machines is less than 100% because the work output is always less than the work input. This is due to the force of friction.

Thus in using a machine to lift an object, the efficiency equation becomes

$$\textbf{Efficiency} = \frac{F_W\,\Delta h}{F_A\,\Delta d}$$

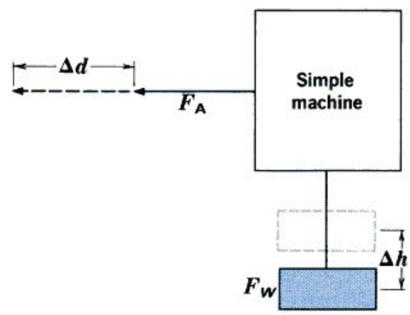


Figure 6-6. Principle of the simple machine. In the absence of friction, the work output, $F_W\,\Delta h$, is equal to the work input, $F_A\,\Delta d$.

where F_W is the weight of the object, Δh is the height through which it is lifted, F_A is the input force applied to the machine, and Δd is the distance through which F_A acts in the direction of the input motion.

The following example illustrates the use of the efficiency equation in solving problems dealing with simple machines.

EXAMPLE A crate is pulled 2 m with constant velocity along an incline that makes an angle of 15° with the horizontal. The coefficient of friction between the crate and the plane is 0.160. Calculate the efficiency that is achieved in this procedure. (Refer to Figure 4-18.)

Given	Unknown	Basic equations
$\mu = 0.160$	Efficiency	$\text{Eff.} = \dfrac{W_{\text{Output}}}{W_{\text{Input}}} = \dfrac{F_W\,\Delta h}{F_A\,\Delta d}$
$\theta = 15°$		$F_f = \mu F_W \cos\theta$
$\Delta d = 2\text{ m}$		$F_P = F_W \sin\theta$

Solution

(a) Finding W_{Output}:
The crate, with a weight of F_W, is lifted through a vertical distance $\Delta h = \Delta d \sin\theta$. Thus,

$$W_{\text{Output}} = F_W \Delta h = F_W \Delta d \sin\theta$$

(b) Finding W_{Input}:
The applied force is the sum of the force of friction, F_f, and the force along the incline, F_P:

$$W_{\text{Input}} = F_A \Delta d = (F_f + F_P)\Delta d$$

Substituting in the remaining Basic equations:

$$W_{\text{Input}} = (\mu F_W \cos\theta + F_W \sin\theta)\Delta d$$

(c) Combining both steps (a) and (b) gives the

Working equation: $$\text{Efficiency} = \frac{F_W\,\Delta d \sin\theta}{F_W\,\Delta d\,(\mu\cos\theta + \sin\theta)}$$

$$= \frac{\sin\theta}{(\mu\cos\theta + \sin\theta)}$$

$$= \frac{\sin 15°}{(0.160)(\cos 15°) + (\sin 15°)}$$

$$= 0.63$$

PRACTICE PROBLEM The efficiency of a pulley system is 73%. The pulleys are used to raise a mass of 58 kg to a height of 3.0 m. What force is exerted on the strand of the pulley if it is pulled for 18.0 m in order to raise the mass to the required height? *Ans.* 130 N

6.5 Definition of Power Like the term *work,* the term *power* has a scientific meaning that differs somewhat from its everyday meaning. When we say a person has great power, we usually mean that the person has great strength or wields great authority. In physics, the term ***power*** means *the time rate of doing work.*

You do the same amount of work whether you climb a flight of stairs in one minute or in five minutes, but your power output is not the same. See Figure 6-7. Power depends upon three factors: the displacement of the object, the force in the direction of the displacement, and the time required.

Since power is the time rate of doing work,

$$P = \frac{W}{\Delta t}$$

where P is power, W is work, and Δt is time. Or, because $W = F\Delta d$ and $v = \Delta d/\Delta t$,

$$P = \frac{F\Delta d}{\Delta t} = Fv$$

When work is measured in joules and time is measured in seconds, power is expressed in *watts* (W). A ***watt*** *is a joule per second.* This unit is named in honor of James Watt, who designed the first practical steam engine. Since the watt is a very small unit, power is more commonly measured in units of 1000 watts, or kilowatts (kW).

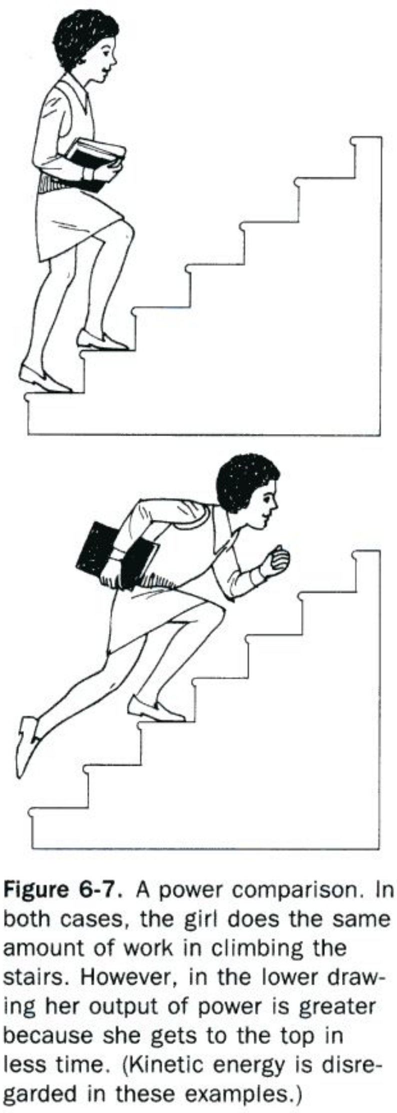


Figure 6-7. A power comparison. In both cases, the girl does the same amount of work in climbing the stairs. However, in the lower drawing her output of power is greater because she gets to the top in less time. (Kinetic energy is disregarded in these examples.)

Figure 6-8. James Watt in his workshop. How many pieces of apparatus can you identify?

The terms watt and kilowatt are used frequently in connection with electricity, but also apply to quantities of power other than electric power. The watt is the unit of power in the metric system and is used to express quantities of mechanical as well as electric power. Another frequently used unit is the "horsepower." It is equal to 746 watts.

EXAMPLE A woman drives her car up a parking ramp 12.0 m high at a constant velocity in 20.0 s. The mass of the car is 1.50 metric tons. Calculate the power output of the car's engine during this time, in horsepower (1 hp = 746 W).

Given	Unknown	Basic equations
$\Delta d = 12.0$ m	P	$P = \dfrac{F_W \, \Delta d}{\Delta t}$
$\Delta t = 20.0$ s		
$m = 1.50 \times 10^3$ kg		$F_W = mg$

Solution

Working equation: $P = \dfrac{mg \, \Delta d}{\Delta t}$

$$= \frac{(1.50 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m})}{(20.0 \text{ s})} \times \frac{(1 \text{ hp})}{(746 \text{ W})}$$

$$= 11.8 \text{ hp}$$

6.6 Power in Rotary Motion By using radian measure, we can compute the power involved in rotary motion the same way we calculated the work done in rotary motion in Section 6.3. Substituting the expression for work in rotary motion, $T\Delta\theta$, the power equation becomes

$$\boldsymbol{P = \frac{T\Delta\theta}{\Delta t}}$$

We saw in Section 5.6 that the time rate of angular displacement, $\Delta\theta/\Delta t$, is called the angular velocity, ω, so

$$P = T\omega$$

The power required to maintain rotary motion against an opposing torque is the product of the torque maintaining the rotary motion and the constant angular velocity.

QUESTIONS: GROUP A

1. Explain whether *work*, in the physics sense, is being done on a suitcase when you (a) pick it up from the floor, (b) carry it at a steady speed on a level street to the bus stop, (c) hold it above the ground while you wait for the bus, and (d) board the bus with the suitcase.
2. What represents the work done when a graph of force versus displacement is constructed?
3. What are three ways to determine the work that is done when a force-versus-displacement graph is not a straight line?
4. If you were to use a machine to increase the produced force, what factor would have to be sacrificed? Give an example.
5. (a) In calculating the work done in rotary motion, what is the expression that takes the place of Δd? (b) What is the equation used in finding the amount of rotary work?
6. For an object moving at a constant speed, list the two expressions for determining the object's power.

GROUP B

7. (a) Using Figure 6-3, determine the average force needed to stretch the spring a distance of 4.0 m. (b) How much work is done in stretching the spring 4.0 m?
8. Use the graph of force versus displacement shown in Figure 6-3 to derive an equation for the work done by the spring.
9. If a machine cannot multiply the amount of work, what is the advantage of using such a machine?
10. A heavy football player climbs a flight of stairs. Halfway up the stairs, a member of the girls' track team rushes past him. Is it possible for both to develop the same amount of power in climbing up the stairs?
11. Find the horsepower rating on a lawn mower or an electric tool. Convert this amount to units of kilowatts.

PROBLEMS: GROUP A

1. A weight lifter heaves a 200.0-kg barbell from the floor to a position directly over his head. If the distance from the floor to his extended arms is 2.50 m, how much work has the weight lifter done?
2. (a) How much work is done in lifting a 750-kg piano vertically 3.0 m to a large set of doors? (b) How much work would be done if the piano was pushed up a frictionless inclined plane to the same set of doors? (c) If the inclined plane was 5.0 m long, how much force would have been needed?
3. How much work is done in pushing a 45.5-kg wooden trunk a distance of 9.75 m across the floor if the coefficient of friction is 0.250?
4. Calculate the work done when a sled is pulled 20.0 m by a force of 105 N exerted on a rope that makes an angle of 50.0° with the horizontal.
5. A pulley system is used to lift the piano mentioned in Problem 2. If a force of 2.0×10^3 N is applied to the piano, and as a result the rope is pulled in 14 m, what is the efficiency of the machine?
6. How much power does a 63.0-kg athlete develop as he climbs a 5.20-m rope in 3.50 s?
7. A 45.0-kg cyclist exerts her full weight on the pedal with each stroke. How much work is done during 100.0 revolutions of the pedals as they turn in a 30.0-cm radius?
8. What is the power rating in kilowatts of a 1.20×10^3-kg elevator that moves

3.50 m from one floor to the one above it in 4.30 s?

9. A 23.0-cm screwdriver is to be used to pry open a can of paint. If the fulcrum is 2.00 cm from the end of the blade and a force of 84.3 N is exerted at the end of the handle, what force is applied to the lid?
10. A pulley system has an efficiency of 87.5%. How much of the rope must be pulled in if a force of 648 N is needed to lift a 105-kg desk 2.46 m?
11. A 175-N bucket of water is to be lifted from the bottom to the top of a 7.30-m well. If a force of 42.0 N is applied at the end of the 36.3-cm handle, how many times must the handle be turned to accomplish this?
12. A 0.50-kW motor moves a lawn tractor at a constant 1.2 m/s. What force is being applied to the tractor?

GROUP B

13. A force of 25.0 N is applied to a 4.50-kg object that is initially at rest. (a) How much work is done during the first 3.00 s of its motion? (b) How much power is developed during this same period of time?
14. A 65-kg crate is pushed at a constant speed up a 3.6-m plane inclined at 24° above the horizontal. If the coefficient of friction is 0.17, how much work is done?
15. A 175-kg flywheel is a uniform disk 1.80 m in diameter. (a) How much work is required to bring it from rest to 94.0 rev/min in 2.00 min? (b) What is the machine's power rating in kilowatts?
16. An elevator motor is rated at 25.0 kW. At what speed could the motor lift an 850.0-kg elevator with three passengers whose masses are 24.3 kg, 45.0 kg, and 64.0 kg?
17. What power must the engine of a 1680-kg car develop to move at a constant speed of 24.5 m/s up a 15° incline if the coefficient of friction between the tires and road is 0.090 0?
18. A 35.4-kg box falls off a truck moving at 40.0 km/h. The box slides to a stop after a distance of 17.5 m. Calculate (a) the force of friction on the box, (b) the work done in stopping it, and (c) the coefficient of friction between the box and pavement.
19. How much work is done in pushing an 85.4-kg grocery cart 2.05×10^2 m if a force is applied at a 40° angle to the horizontal and the coefficient of friction between the wheels and the floor is 0.025 0?

ENERGY

Potential and kinetic energy can each have a variety of forms.

6.7 Gravitational Potential Energy In Chapter 1 we saw that there are two kinds of energy, potential and kinetic. In the following sections, we will deal quantitatively with these concepts and see how the various forms of energy are expressed in terms of work units.

The potential energy acquired by an object equals the work done against gravity or other forces to place it in position, as shown in Figure 6-9. As we saw in Section 6.1, the equation for calculating work when the force acts in the same direction as the displacement is

$$W = F\Delta d$$

Therefore the equation for potential energy is

$$E_p = F\Delta d$$

In lifting an object, F is its weight, which from Newton's second law of motion equals mg, and Δd is the vertical distance Δh through which it is lifted. Hence the potential energy equation can be written

$$E_p = mg\Delta h$$

The *gravitational potential energy* defined by this equation is expressed in relation to an arbitrary reference level where $h = 0$. The reference level is usually determined by the nature of the problem. Sea level, street level, ground level, or floor level are all commonly used reference levels.

When the mass of an object is given in kilograms, the height in meters, and the acceleration due to gravity in m/s^2, the gravitational potential energy is expressed in joules. Thus if a $5\bar{0}$-kg mass of steel is raised 5.0 m, its gravitational potential energy is

$$E_p = mg\Delta h$$

$$E_p = 5\bar{0}\ \text{kg} \times 9.8\ \text{m/s}^2 \times 5.0\ \text{m}$$

$$E_p = 2.5 \times 10^3\ \text{J}$$

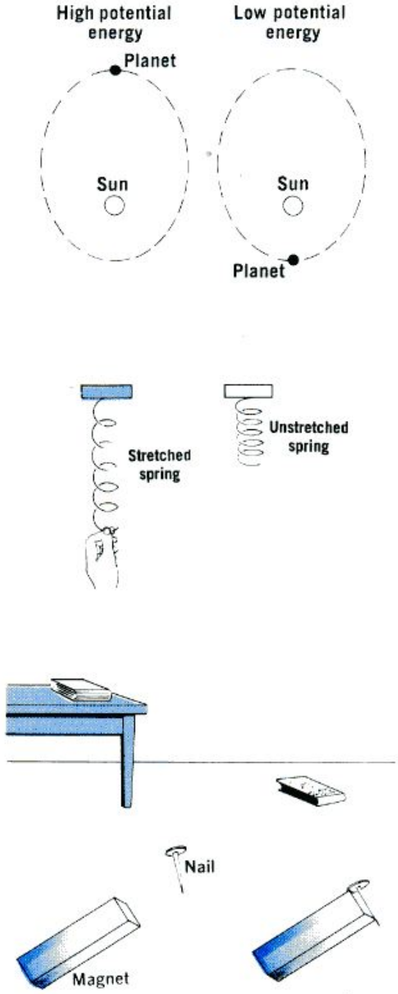


Figure 6-9. Examples of potential energy. Can you identify the reference level for each pair of situations?

6.8 Kinetic Energy in Linear Motion As we saw in Section 3.7, the velocity of a freely falling object that starts from rest, expressed in terms of the acceleration of gravity and the distance traveled, is given by the equation

$$v = \sqrt{2g\Delta d}$$

Solving for Δd, we obtain

$$\Delta d = \frac{v^2}{2g}$$

Since Δd in this equation corresponds to Δh in the equation $E_p = mg\Delta h$, let us substitute the expression we have just derived for Δd in place of Δh. The equation for the kinetic energy of a moving object then becomes

$$E_K = mg \times \frac{v^2}{2g}$$

$$E_K = \tfrac{1}{2}mv^2$$

Although this equation for kinetic energy was derived from the motion of a falling body, it applies to motion in any direction or from any cause.

As in the case of gravitational potential energy, kinetic energy is expressed in joules if the mass is given in kilograms and the velocity in meters per second. Thus, if a baseball has a mass of 0.14 kg and it's thrown with a velocity of 26 m/s, its kinetic energy is

$$E_K = \tfrac{1}{2}mv^2$$

$$E_K = \frac{0.14 \text{ kg } (26 \text{ m/s})^2}{2}$$

$$E_K = 47 \text{ J}$$

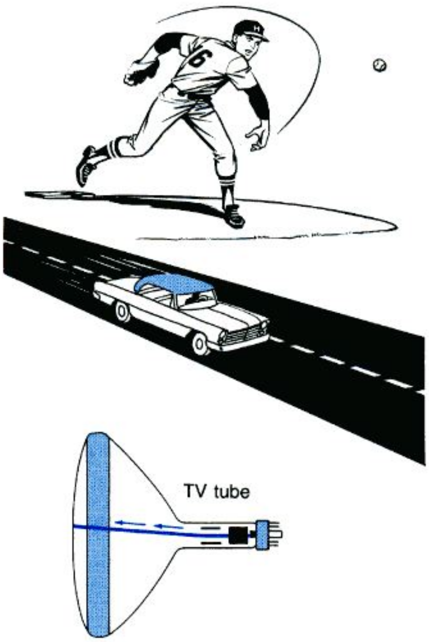


Figure 6-10. Examples of kinetic energy. What is the frame of reference for the velocity of the moving object in each case?

6.9 Kinetic Energy in Rotary Motion As we saw in Section 5.6, the angular velocity, ω, of a rotating body that starts from rest is

$$\omega = \sqrt{2\alpha\Delta\theta}$$

where α is the angular acceleration and $\Delta\theta$ is the angular displacement. The equation given in Section 5.8 for the relationship among torque, rotational inertia, and angular acceleration is

$$T = I\alpha$$

Solving both of these expressions for angular acceleration,

$$\alpha = \frac{\omega^2}{2\Delta\theta} \text{ and } \alpha = \frac{T}{I}$$

Setting these two expressions equal,

$$\frac{T}{I} = \frac{\omega^2}{2\Delta\theta}$$

Solving for $T\Delta\theta$,

$$T\Delta\theta = \frac{I\omega^2}{2}$$

When only the net force and torque are considered in these equations, all of the work done to produce rotation appears as kinetic energy. Since for rotary motion $W = T\Delta\theta$, the equation for kinetic energy, E_K, is

$$E_K = \tfrac{1}{2}I\omega^2$$

The wheels of a moving car have both linear and rotational kinetic energy.

The wheel of a moving automobile has both linear motion and rotary motion. The wheel turns on its axle as the axle moves along parallel to the road. The kinetic energy of such an object is the sum of the kinetic energy due to linear motion and the kinetic energy due to rotary motion.

$$E_K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

EXAMPLE A metal disk with a mass of 1.30 kg rolls along a horizontal floor with a constant velocity of 1.68 m/s. Calculate the kinetic energy of the moving disk.

Given	Unknown	Basic equations
$m = 1.30$ kg $v = 1.68$ m/s	E_K	$E_K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $\omega = \Delta\theta/\Delta t$ $v = \Delta d/\Delta t$ $\theta = \Delta d/r$

Solution

From Fig. 5-11 we know that $I = \frac{1}{2}mr^2$.

$$\begin{aligned} \textit{Working equation: } E_K &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{1}{2}mr^2)(v/r)^2 \\ &= \tfrac{3}{4}mv^2 \\ &= \tfrac{3}{4}(1.30 \text{ kg})(1.68 \text{ m/s})^2 \\ &= 2.75 \text{ J} \end{aligned}$$

When energy is supplied to an object and simultaneously gives it both linear and rotary motion, the energy division depends on the rotational inertia of the object. If a ring and a solid disk of equal mass and diameter roll down the same incline, as in Figure 6-11, the disk will accelerate more. Its rotational inertia is less than that of the ring, thus its rotational kinetic energy is less. Its linear kinetic energy is therefore greater than that of the ring; the disk acquires a higher linear velocity and reaches the bottom of the incline first. However, at the bottom of the incline the total kinetic energy of the ring will be the same as that of the disk since both ring and disk had the same gravitational potential energy at the top.

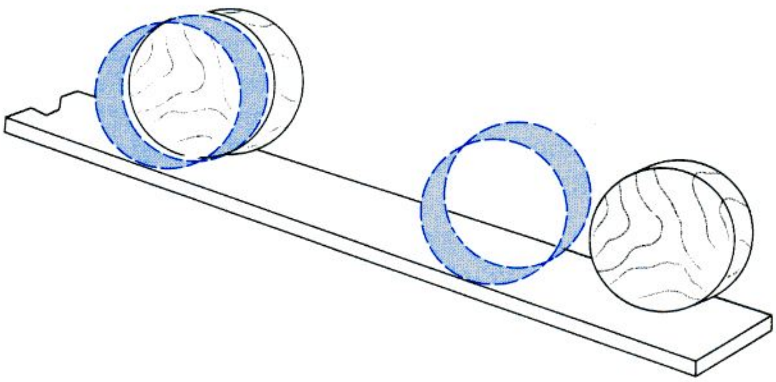


Figure 6-11. A kinetic energy race. If the ring and disk have equal masses and diameters, the rotational inertia of the disk is less than that of the ring. Hence the disk will accelerate more.

Compare Equation 3 with the one for kinetic energy in Section 6.8. Why are they similar?

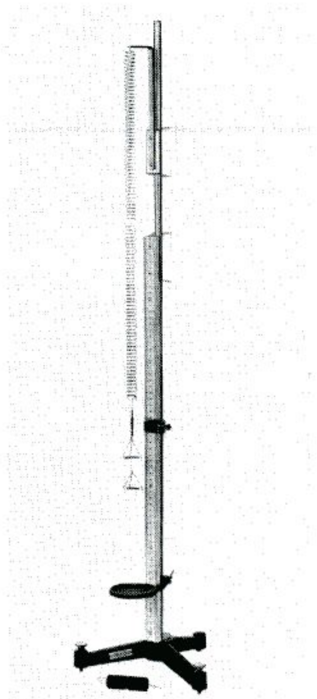


Figure 6-12. Apparatus for measuring the force constant of a spring. If the weight is made to move up and down, the interchange of potential and kinetic energy can also be measured.

6.10 Elastic Potential Energy

When a spring is compressed, energy is stored in the spring. At any instant during the compression, the elastic potential energy in the spring is equal to the work done on the spring. The potential energy in a stretched or compressed elastic object is called ***elastic potential energy.***

The work required to stretch or compress a spring does not depend on the weight of the spring. Consequently, gravity is not involved in the measurement of elastic potential energy. Instead, the work required to stretch or compress a spring is dependent upon a property of the spring known as the *force constant*. The force constant does not change for a specific spring so long as the spring is not permanently distorted.

The force required to stretch a spring is written as

$$F = k\Delta d \qquad \text{(Equation 1)}$$

where k is the force constant of the spring and Δd is the distance over which F is applied. We noted in Section 6.2 that the force is directly proportional to the amount of stretching (within limits). Consequently, the work done on the spring also varies directly with the amount of stretching. The total amount of work, therefore, is given by the equation.

$$W = \tfrac{1}{2}F\Delta d \qquad \text{(Equation 2)}$$

where F is the force exerted on the spring at the end of the stretch through distance Δd. Because the force varies from zero to F, the equation gives the average of these two values, or $\frac{1}{2}F$.

Substituting the expression for F from Equation 1 in Equation 2, we get

$$W = \tfrac{1}{2}k\,(\Delta d)^2 \qquad \text{(Equation 3)}$$

This equation applies to the compression of a spring as well as the stretching of a spring. Since the potential energy of an object is equal to the work done on the object, Equation 3 can be rewritten as a potential energy equation

$$E_p = \tfrac{1}{2}k\,(\Delta d)^2$$

where E_p is the elastic potential energy. This equation represents ideal conditions. In actual practice, a small fraction of the work of stretching or compression is converted into heat energy in the spring and does not show up as elastic potential energy.

6.11 Conservation of Mechanical Energy

As we saw in Section 5.10, the vibration of a mass on a spring and the swinging of a pendulum are both examples of simple harmonic motion. Both can also be used to illustrate an important principle of physics called the ***law of conservation of mechanical energy.*** This law states that *the sum of the potential and kinetic energy of an energy system remains constant when no dissipative forces act on the system.*

This is a special case of the law of conservation of energy discussed in Section 1.13.

Gravitational forces and elastic forces are called *conservative forces* because they conform to the law of conservation of mechanical energy. There are forces, however, that produce deviations from the law of conservation of mechanical energy. The force of friction is an example. Forces of this type are called nonconservative, or *dissipative forces.*

The reason that friction is a dissipative force is that it produces a form of energy (heat) that is not mechanical. Energy is lost to the system. When considered in light of the more comprehensive law of conservation of *total* energy, there is no "lost" energy, of course. The calculation of heat energy and its role in energy transformations will be discussed in Chapter 8.

Another way to distinguish between conservative and dissipative forces is to observe the relationship between the force and the path over which it acts. In the case of a conservative force, the work done and energy involved are completely independent of the length of the path, provided the paths have the same end point. For example, in the illustration of gravitational potential energy in Figure 6-13, the gravitational potential energy of both men is the same, provided they have equal masses, even though one of them travels a greater distance than the other one. The mass of the men and their height above the reference level are the only factors required for the calculation of gravitational potential energy.

The amount of heat energy lost through friction, a dissipative force, can be quite different for two objects moving through the same height, however. Where the path is longer, the amount of mechanical energy that is converted to heat energy will be greater. This is true even if the frictional forces remain constant. Thus the work done by a given dissipative force varies directly with the length of the object path.

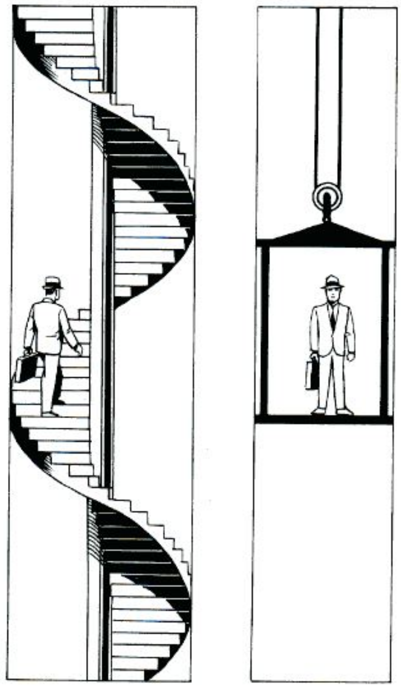


Figure 6-13. The work done by conservative forces is independent of the path. The potential energy of the man is, in each case, dependent only on his mass and his distance above the reference level.

QUESTIONS: GROUP A

1. What forms of energy are present in the following situations? (a) A diver who stands on the edge of a 10-m platform. (b) A bowstring that has been pulled back ready to launch an arrow. (c) A student who climbs the stairs to the school's library. (d) A penny that is dropped from the second floor. (e) The wheel of a wagon that is rotating as the wagon is pulled up a hill.

2. How does rotary kinetic energy differ from linear kinetic energy?
3. What would be the shape of a graph that shows an object's kinetic energy as a function of the object's speed? Be specific.

GROUP B

4. The kinetic and gravitational potential energy of a nail being driven into a piece of wood change very little. (a) What happens to the work done by the hammer? (b) Does this violate the law of conservation of energy?
5. Describe how the energy changes as a pole vaulter approaches the bar, clears the bar, and lands on the cushions below the bar.

PROBLEMS: GROUP A

1. A 65-kg diver is poised at the edge of a 10.0-m platform. Calculate the diver's gravitational potential energy relative to the pool.
2. What is the linear kinetic energy of a 1250-kg car moving at 45.0 km/h?
3. The force constant of a spring in a child's toy car is 550 N/m. How much elastic potential energy is in the spring if it is compressed a distance of 1.2 cm?
4. (a) What is the potential energy of a 1050-N rock on the edge of a cliff that is 20.4 m high? (b) If the rock falls, what is its kinetic energy when it strikes the ground? (c) How fast is it moving when it strikes the ground?
5. Calculate the rotary kinetic energy of a 32-cm diameter bicycle wheel with a mass of 5.5-kg as it spins at 65 rev/min.
6. A force of 22 N is exerted horizontally on an 18-kg box to move it 7.6 m across the floor. If the box was initially at rest and is now moving at 3.2 m/s, calculate (a) the work done, (b) the final kinetic energy of the box, and (c) the energy converted to thermal energy due to friction.
7. A 72-kg pole vaulter running at 8.4 m/s completes a vault. If all of his kinetic energy is transformed into gravitational potential energy, what is the maximum height of the bar?
8. To cut down on injuries, a highway guardrail is designed to be moved a maximum of 5.00 cm when struck by a car. What is the minimum force constant of the material in the guardrail if it is to withstand the impact of a 1250-kg car moving at 15.0 km/h?
9. (a) Calculate the rotary kinetic energy of the earth. (b) What is the earth's average linear kinetic energy as it orbits the sun?

GROUP B

10. A 3.00-kg ball rolls up a 45° incline. (a) If the ball is moving at 5.00 m/s at the bottom, what is its initial rotary and linear kinetic energy? (Ignore the effect of friction.) (b) How far does the ball roll before it stops?
11. A 100.0-g arrow is pulled back 30.0 cm against a bowstring. If the force constant of the bow and string is 1250 N/m, at what speed will the arrow leave the bow?
12. A 350.0-kg roller coaster car is poised at the top of a 42.0-m hill. (a) How fast will the car be moving at the bottom of the incline? (b) As it goes over the top of the next hill, 30.0 m high? (c) Could this problem be done without knowing the mass of the car?

PHYSICS ACTIVITY

Obtain two soup cans of equal mass and size, one having solid contents and the other loose. Hold the cans at the top of an inclined plank and release them simultaneously. Which one reaches the bottom first? Why?

MOMENTUM

6.12 *The Nature of Momentum* More force is needed to stop a train than to stop a car, even when both are moving with the same velocity. A bullet fired from a gun has more penetrating power than a bullet thrown by hand, even when both bullets have the same mass. The physical quantity that describes this aspect of the motion of an object is called *momentum*. ***Momentum** is the product of the mass of a moving body and its velocity.* The equation for momentum is

$$p = mv$$

where p is the momentum, m is the mass, and v is the velocity of an object.

In the example of the car and train, the greater mass of the train gives it more momentum than the car. Consequently, a greater change of momentum is involved in stopping the train than in stopping the car. In the case of the bullets, the greater momentum of the fired bullet is due to its greater velocity; a large change of momentum takes place when the speeding bullet is stopped.

From Newton's second law of motion, we can derive an important relationship involving momentum. In Section 3.5 we saw that the equation for average acceleration is $a_{av} = \Delta v/\Delta t$. When we substitute this value of a in the equation for Newton's second law, $F = ma$, we get

$$F = \frac{m\Delta v}{\Delta t} \quad \text{or} \quad F\Delta t = m\Delta v$$

The first of these equations states that force is the time rate of change of momentum. The second equation states that impulse equals the change in momentum.

*The product of a force and the time interval during which it acts, $F\Delta t$, is called **impulse**.* Hence from Newton's second law of motion we have established that impulse equals change in momentum. The equation $F = m\Delta v/\Delta t$ tells us that when a force is applied to a body, the body's rate of change of momentum is equal to the force. Since the equation is a vector equation, we also know that the body's rate of change of momentum is in the direction of the force.

A good example of the relationship between impulse and change in momentum is a bat hitting a baseball. See Figure 6-14. The impulse imparted to the ball depends on the force with which the ball is hit and the length of time during which the ball and bat are in contact. On leaving the bat, the ball has acquired a momentum equal to the product of its mass and its change of velocity. In a sense, the impulse produced the change of momentum; hence the two are equal.

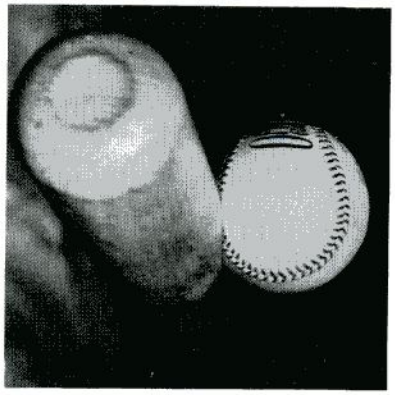


Figure 6-14. During the time of contact, much of the momentum of the bat is transferred to the baseball.

6.13 The Conservation of Momentum In Figure 6-15, a boy with a mass of $4\bar{0}$ kg and a man with a mass of $8\bar{0}$ kg are standing on a frictionless surface. When the man pushes on the boy from the back, the boy moves forward and the man moves backward.

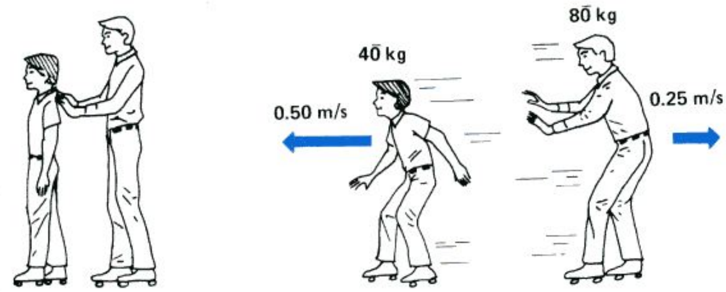


Figure 6-15. Conservation of momentum. On a frictionless surface, the momentum of the boy toward the left is equal to the momentum of the man toward the right.

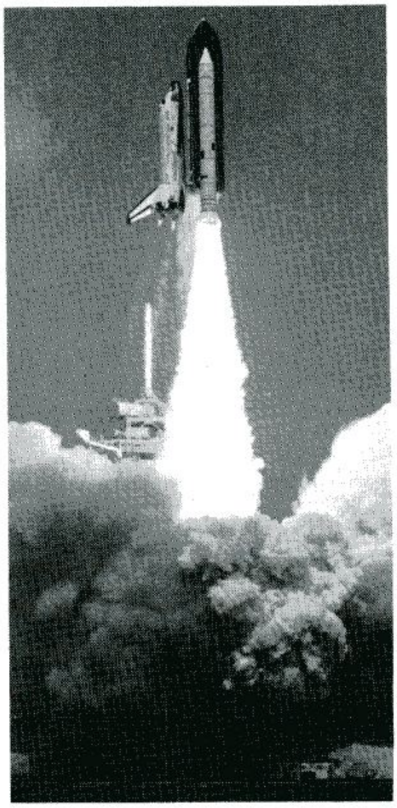


Figure 6-16. Conservation of momentum in a rocket. The Space Shuttle is launched when momentum equal in magnitude to that of the exhaust gases is imparted to the space vehicle.

The velocities with which the boy and the man move are specified by one of the most important principles of physics, the ***law of conservation of momentum.*** This law states that *when no net external forces are acting on a system of objects, the total vector momentum of the system remains constant.*

Let us apply this law to the situation in Figure 6-15. Initially the man and the boy are at rest. The system, therefore, has zero momentum. When the man and the boy move apart, the law of conservation of momentum requires that the total vector momentum remains zero. Hence the momentum of the boy in one direction must equal that of the man in the other direction.

If the boy moves with a velocity of 0.50 m/s, the man will move with a velocity of 0.25 m/s since the mass of the man is twice as great as that of the boy. The momentum of the boy in one direction ($4\bar{0}$ kg $\times$ 0.50 m/s) must equal the momentum of the man in the opposite direction ($8\bar{0}$ kg $\times$ 0.25 m/s).

An important application of the law of conservation of momentum is the launching of a rocket. When a rocket fires, hot exhaust gases are expelled through the rocket nozzle. The gas particles have a momentum equal to the mass of the particles multiplied by their exhaust velocity. Momentum equal in magnitude is therefore imparted to the rocket in the opposite direction. See Figure 6-16. Newton's third law of motion (Section 3.10) is a special case of the law of conservation of momentum.

6.14 Inelastic Collisions The law of conservation of momentum is very helpful in studying the motions of colliding objects. Collisions can take place in various ways, and we shall see how the momentum conservation principles apply in several such cases.

In Figure 6-17(A), two carts of equal mass approach each other with velocities of equal magnitude. A lump of putty is attached to the front of each chart so that the two carts will stick together after the impact. This situation is an example of ***inelastic collision.*** Since the carts are traveling along the same straight line, it is also an example of a *collision in one dimension.*

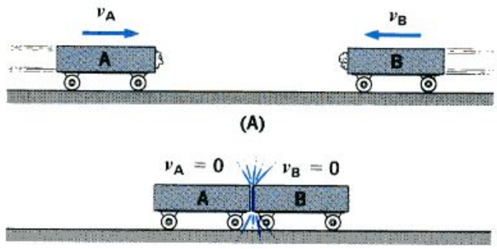


Figure 6-17. An inelastic collision. The carts have equal masses and approach each other with velocities of equal magnitude along the same straight line. The total momentum of the system is the same before and after the collision.

The momentum of cart **A** is $m_A v_A$. It is equal in magnitude to the momentum of cart **B**, $m_B v_B$. However, the direction of v_A is opposite to the direction of v_B, so

$$v_A = -v_B$$

Consequently, $$m_A v_A = -m_B v_B$$

and $$m_A v_A + m_B v_B = 0$$

This means that the total vector momentum of the system of two moving carts is zero. (We assume that the system is *isolated,* that is, there are no net external forces acting on it. In actual collision studies, the external force of friction is usually minimized by using rolling carts or air tracks, such as the one shown in Figure 6-18.)

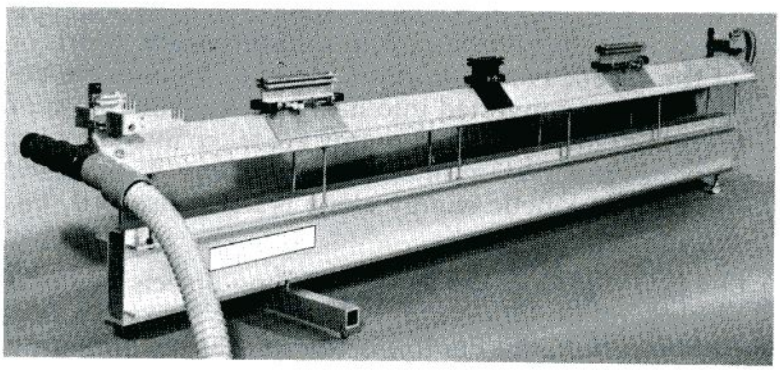


Figure 6-18. An air track designed for collision experiments. Air jets minimize the friction between the track and the masses placed on it.

After the carts in Figure 6-17(B) collide, they both come to rest. The cart velocities, v_A and v_B, are now both zero; the sum of the momenta of the two carts is zero, just as it was when the carts were in motion in opposite directions. Thus the total vector momentum of the system is unchanged by the collision.

If one of the carts has a greater mass than the other, although its velocity is still of equal magnitude but opposite sign, the outcome of the collision is different. After impact, the combined carts will move in the direction of the cart with the larger mass. The velocity of the combined carts will be such that the total momentum of the system remains unchanged.

For example, suppose

$$m_A = 2m_B$$

and $\quad v_A = -v_B$, as before.

Then, $\quad m_A v_A + m_B v_B \neq 0$

Since $\quad m_B = \dfrac{m_A}{2}$, by substitution,

$$m_A v_A - \tfrac{1}{2} m_A v_A = \tfrac{1}{2} m_A v_A$$

This means that the total momentum of the system, before and after the collision, is $\frac{1}{2}m_A v_A$; the combined carts will move with this momentum in the direction of the original velocity of cart **A.** The velocity after the collision can be found by dividing the total momentum by the total mass

$$\frac{\frac{1}{2} m_A v_A}{\frac{3}{2} m_A} = \tfrac{1}{3} v_A$$

Kinetic energy is not conserved in an inelastic collision.

When carts of equal and opposite momenta collide inelastically, they come to rest. Before the collision they have kinetic energy; after the collision, they do not. This is typical of all inelastic and partially elastic collisions. Much or all of the kinetic energy that the moving objects have before collision is converted into heat or some other form of energy. If all these forms of energy are taken into account, the law of conservation of energy holds true for inelastic collisions. But kinetic energy alone is not conserved.

6.15 Elastic Collisions When colliding objects rebound from each other without a loss of kinetic energy, a perfectly ***elastic collision*** has just occurred. The only perfectly elastic collisions occur between atomic and subatomic particles. The situation can be approximated, however, by the use of hard steel balls or springs on an air track. The momentum conservation law holds for elastic collisions as well as inelastic ones and for collisions that are partly elastic and partly inelastic.

The law also holds for *collisions in two dimensions,* that is, when the colliding objects meet at an angle other than head-on. Figure 6-19(A) shows the elastic, two-dimensional collision of two balls of equal mass. Ball **B** collides at

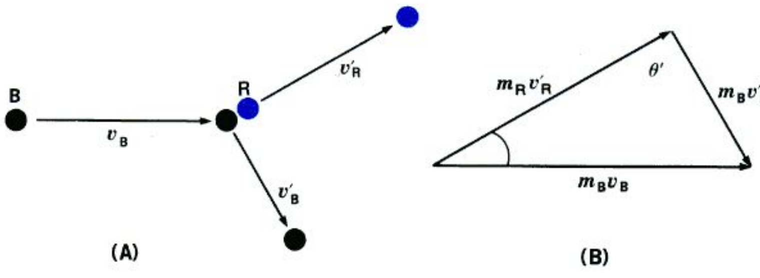


Figure 6-19. A vector diagram can be used to show that the total momentum and kinetic energy of a system both remain constant in a two-dimensional collision.

an angle with ball **R**, which is initially at rest. Figure 6-19(B) is the vector diagram representing the momenta of the balls before and after collision.

Review the parallelogram method of adding vectors explained in Section 2.11.

In Figure 6-19(B), $m_B v_B$ is the momentum of the black ball before the collision. Since the blue ball is stationary before the collision, its momentum is zero and is not represented by a vector. The total momentum of the system before collision is

$$\mathbf{m_B v_B + 0 = m_B v_B}$$

After the collision, the measured velocities of the black and blue balls are v'_B and v'_R respectively, in the directions indicated. The total momentum now is the vector sum $m_B v'_B + m_R v'_R$. Since the momentum is conserved, the vector $m_B v_B$ must equal the vector sum of $m_B v'_B$ and $m_R v'_R$. The vector diagram appears as a closed triangle.

Since kinetic energy is also conserved in Figure 6-19,

$$\tfrac{1}{2} m_B {v_B}^2 = \tfrac{1}{2} m_B {v'_B}^2 + \tfrac{1}{2} m_R {v'_R}^2$$

Since the two balls have equal masses, this equation can be simplified to

$$ {v_B}^2 = {v'_B}^2 + {v'_R}^2$$

This is the equation that relates the sides and hypotenuse of a right triangle, as in Figure 6-19(B). So it follows from the laws of conservation of energy and momentum that, when a moving ball strikes a stationary ball of equal mass other than head-on in an elastic collision, the two balls move away from each other at right angles.

The conservation of momentum also holds for partially elastic collisions, for collisions involving more than two bodies, and for three-dimensional situations. The following examlpes show how to solve collision problems.

EXAMPLE A 1.20-kg cart that has an eastward velocity of 0.50 m/s collides head-on inelastically with a 1.60-kg cart having a westward velocity of 0.70 m/s. Disregarding the slowing-down effects of friction, calculate the new velocity (speed and direction) of the two-cart system.

Given	Unknown	Basic equation
$m_E = 1.20$ kg $v_E = 0.50$ m/s $m_W = 1.60$ kg $v_W = -0.70$ m/s	v_{E+W}	$m_E v_E + m_W v_W = m_{E+W} v_{E+W}$

Solution

$$\text{Working equation: } v_{E+W} = \frac{m_E v_E + m_W v_W}{m_{E+W}}$$

$$= \frac{(1.20 \text{ kg})(0.50 \text{ m/s}) + (1.60 \text{ kg})(-0.70 \text{ m/s})}{(1.20 \text{ kg} + 1.60 \text{ kg})}$$

$$= -0.19 \text{ m/s (minus sign indicates westward motion)}$$

EXAMPLE A ball moving eastward with a speed of 0.70 m/s hits a stationary ball of equal mass in an elastic collision. After the collision, the second ball moves away in a direction $3\bar{0}°$ north of east. Calculate the speed of this ball. (Refer to Figure 6-19(B).)

Given	Unknown	Basic equation
$m_B = m_R$ $v_B = 0.70$ m/s $\theta = 3\bar{0}°$ $\theta' = 9\bar{0}°$	v'_R	$m_R v'_R = m_B v_B \cos\theta$

Solution

$$\text{Working equation: } v'_R = v_B \cos\theta$$

$$= (0.70 \text{ m/s})(\cos 3\bar{0}°)$$

$$= 0.61 \text{ m/s}$$

6.16 Angular Momentum For rotary motion, the relationship between impulse and the change of angular momentum is similar to that for linear motion. Using the symbols for rotary motion, the equation becomes

$$\boldsymbol{T\Delta t = I\omega_f - I\omega_i}$$

where $T\Delta t$ is the ***angular impulse*** and $I\omega_f - I\omega_i$ is the change in ***angular momentum.*** The dimensions of both angular impulse and angular momentum are $kg \cdot m^2/s$.

Just as the linear momentum of an object is unchanged unless a net external force acts on it, *the angular momentum of an object is unchanged unless a net external torque acts on it.* This is a statement of the *law of conservation of angular momentum.* A rotating flywheel, which helps maintain a constant angular velocity of the crankshaft of an automobile engine, is an illustration. The rotational inertia of a flywheel is large. Consequently torques acting on it do not produce rapid changes in its angular momentum. As the torque produced by the combustion in each cylinder tends to accelerate the crankshaft, the rotational inertia of the flywheel resists this action. Similarly, as the torques produced in the cylinders where compression is occurring tend to decelerate the crankshaft, the rotational inertia of the flywheel resists this action and the flywheel tends to maintain a uniform rate of crankshaft rotation.

If the distribution of mass of a rotating object is changed, its angular velocity changes so that the angular momentum remains constant. A skater spinning on the ice with arms folded, as in Figure 6-20, turns with relatively constant angular velocity. If she extends her arms, her rotational inertia increases. Since angular momentum is conserved, her angular velocity must decrease.

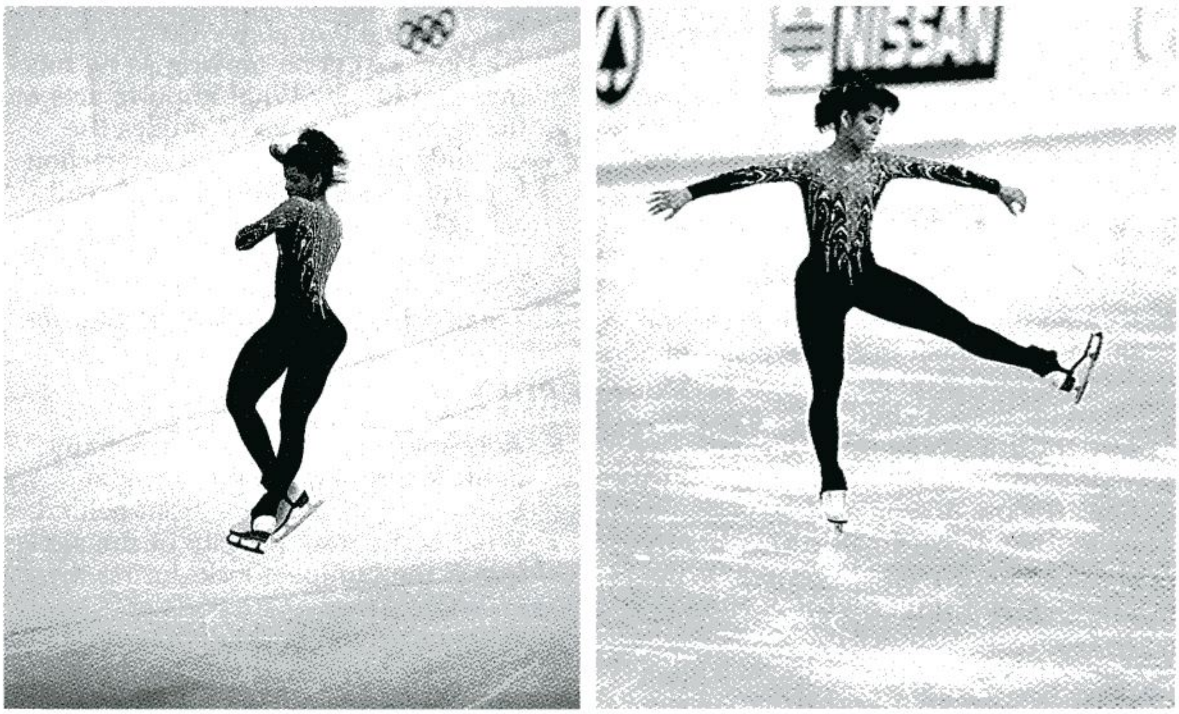


Figure 6-20. Conservation of angular momentum. When world-class figure skater Debi Thomas spins (left), her rotational inertia is small and her rotational velocity is high. When she extends her arms (right), the situation is reversed.

QUESTIONS: GROUP A

1. (a) Define impulse. (b) Define momentum. (c) Are they vectors or scalars? Explain.
2. Mathematically derive the relationship between impulse and momentum.
3. What is the difference between an elastic and an inelastic collision?
4. How does the law of conservation of momentum apply to the launch of a rocket ship?
5. If momentum is conserved, what else must happen when an object is dropped toward the earth?
6. Why does a fielder draw his hand back as he catches a baseball?

GROUP B

7. You are participating in a Physics Olympics event called the egg toss. How could you improve your chances of catching a tossed egg?
8. A diver leaps off a high platform in a layout position. If the diver pulls into a tuck position, what will happen to her rotary speed? Why?
9. What is the function of the long pole carried by a tightrope walker?
10. How do impulse and momentum explain why a 110-kg defensive lineman has more trouble changing direction than a 75-kg quarterback running at the same speed?
11. Can an object with a large mass and one with a small mass have the same momentum? Explain.
12. (a) Why does a child's toy top remain upright if it is spinning, but falls over as it slows down to a stop? (b) Why does it slow down?
13. When one billiard ball strikes another, there are two possible results. What are they and under what circumstances will each occur?
14. The safety net under a trapeze artist is loose. Use your knowledge of impulse and momentum to explain why.
15. Describe the situation in Figure 6-15 if (a) the boy instead of the man does the pushing and (b) the boy and the man push simultaneously.

PROBLEMS: GROUP A

1. An impulse of 20.0 N·s is applied to a 5.0-kg wagon initially at rest. What is its final speed?
2. (a) What impulse is required to stop a 0.250-kg baseball traveling 42.0 m/s? (b) If the ball is in the fielder's mitt for 0.100 s as it is being stopped, what is the average force acting on the ball?
3. A 60.0-g egg moving at 4.8 m/s is caught by a student. (a) If the time of interaction is 0.25 s, what is the average force on the egg? (b) If the maximum force the egg can withstand is 650 N, what minimum time is required to keep the egg intact?
4. A 50.0-kg person jumps from a window ledge 4.0 m above the pavement. (a) How fast is he moving as he hits the ground? (b) What impulse acts on the person's legs as he strikes the ground? (c) If the time of interaction is 0.060 seconds, how much force is acting?
5. The muzzle velocity of a 50.0-g shell leaving a 3.00-kg rifle is 400.0 m/s. What is the recoil velocity of the rifle?

GROUP B

6. A 1250-kg car is stopped at a traffic light. A 3550-kg truck moving at 8.33 m/s strikes the car from behind. What is the new velocity of the system if the bumpers lock during the collision?
7. A 65-kg person is skiing down a hill. The skier's speed at the bottom is 15 m/s. If the skier hits a snowdrift

and stops in 0.30 s, (a) how far does she go into the drift? (b) With what average force will she strike the drift?

8. Calculate the angular momentum of the rotating earth.
9. A 25-kg wagon moves eastward at 3.5 m/s. A force acting on the wagon for 4.0 s gives it a speed of 1.3 m/s to the west. Calculate (a) the impulse acting on the wagon and (b) the magnitude and direction of the force.
10. A 55.0-kg sailor jumps from a dock into a 100.0-kg rowboat at rest beside it. If the linear velocity of the sailor is 5.00 m/s as he leaves the dock, what is the resultant velocity of the sailor and the boat?
11. In the multiple-exposure photograph in Figure 6-21, a large ball approaches from the top and a smaller one from the bottom. The mass of the large ball is 150 g. The photo shows the balls at equal time interals. By means of a vector diagram, find the mass of the smaller ball.
12. An 85.0-g bullet is shot at a 3.00 kg piece of wood at rest at the edge of a counter 1.20 m high. If the bullet becomes embedded in the block and they land 5.0 m from the counter, what was the initial speed of the bullet?
13. A bowling ball of mass 8.00 kg, moving at 2.00 m/s collides with an identical ball at rest. If the first ball moves off at 30.0° from its original path, what are the speeds of the balls as they separate?

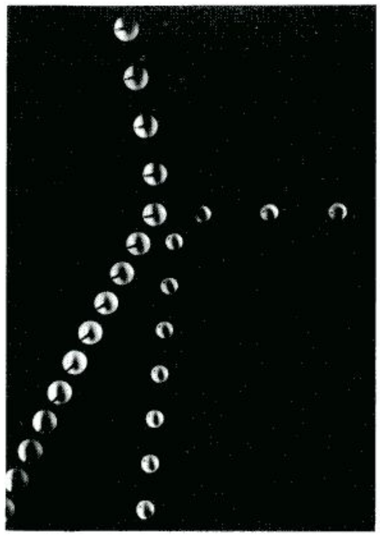


Figure 6-21.

SUMMARY

Work is the product of a displacement and the component of the force in the direction of the displacement. The unit of work is the joule. Work done by varying forces can be found by calculating the area under the curve of a graph in which the horizontal axis denotes the displacement and the vertical axis denotes the force. Radian measure is used to compute work done in rotary motion.

Machines can be used to multiply force at the expense of distance. The efficiency of a machine is the ratio of the useful work output to the total work input and is expressed as a percentage. Power is the time rate of doing work. It is measured in watts. Radian measure is used to compute power in rotary motion.

Gravitational potential energy is equal to the work done against gravity to place an object in position. Thus energy is measured in units of work. The kinetic energy of a body moving in a straight line is directly proportional to the mass of the body and the square of its velocity. In rotary motion, the kinetic energy is directly proportional to the rotational inertia and the square of the angular velocity.

Elastic potential energy is directly proportional to the force constant and the square of the amount of stretch. The law of conservation of mechanical energy states that the sum of the potential and kinetic energy of an ideal (friction-free) energy system remains constant.

Momentum is the product of the mass of a moving body and its velocity. The change of momentum of a moving body is equal to its impulse, which is the product of a force and the time interval during which the force acts. The momentum of a system of objects is conserved when no net external forces are acting on the system. This phenomenon is an example of the law of conservation of momentum. The reaction principle is an application of this law. The conservation of momentum also helps to describe the motion of objects colliding elastically or inelastically and in one, two, or three dimensions. Unless a net external torque acts on a rotating object, its angular momentum is also conserved.

VOCABULARY

angular impulse
angular momentum
efficiency
elastic collision
elastic potential energy
gravitational potential energy
impulse
inelastic collision
joule
law of conservation of mechanical energy
law of conservation of momentum
machine
momentum
power
watt
work