MAGNETIC FORCES

Magnetic fields exert forces on moving charges. If these charges are confined to wires then the fields can be thought of as exerting forces on current-carrying wires. The vector equation for the force is

$$F = il \times B \tag{34-1}$$

where l is the length of the wire taken in the direction of the current. In most instances it is not necessary to do a formal vector cross product because the direction of the current and the magnetic field are perpendicular. The one instance where this becomes important is in considering the torque on a current-carrying wire. This problem is central to several topics and is taken up as a separate exercise.

A Current-Carrying Loop in a *B* Field

The study of torque on a current-carrying loop leads to the definition of the magnetic moment of a circulating current, the understanding of the meter movement (voltmeter and current meter), and electric motors and generators. A loop of wire placed in a magnetic field and oriented as shown in Fig. 34-3 will have no torque or maximum torque, depending on the orientation (rotation) of the loop with respect to the magnetic field.

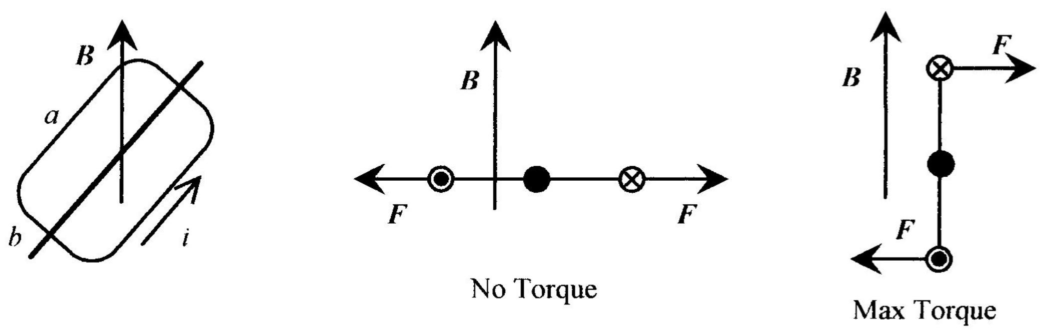

Fig. 34-3

As shown in the middle diagram the piece of wire on the right carrying a current, i, represented by the tail of the current arrow, has a force to the right ($l \times B$) producing no torque. The piece of wire on the left produces a force to the left also producing no torque. In the diagram on the right the forces are in the same direction but for this orientation of the coil the torque is a maximum.

Figure 34-4 shows the torque for angles other than for zero or maximum torque. The torque is the component of this force at right angles to the line connecting the two sides of the coil times $b/2$ (the lever arm) times 2 (two wires).

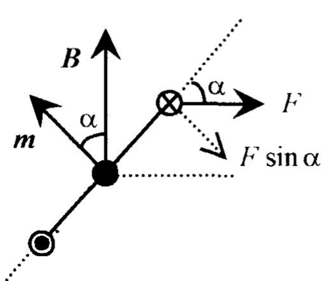

Fig. 34-4

The appropriate component of the force is $F\sin\alpha$. The vector, ***m***, the **magnetic moment**, is normal to the plane of the coil and points in the direction of the thumb when the fingers are curled naturally in the direction of the current in the coil. The angle between ***m*** and ***B*** is the same as the angle between force and lever arm.

$$\tau = 2(b/2)iaB\sin\alpha = iAB\sin\alpha$$

The product ab is the area, A, of the loop. The $\sin\alpha$ term implies a cross product and if iA is taken as the magnitude of ***m***, the **magnetic moment,** with the direction as defined in Fig. 34-4 then the vector equation for the torque is

$$\tau = \boldsymbol{m} \times \boldsymbol{B} \quad \text{or} \quad \tau = mB\sin\alpha \quad \text{with direction defined by } \boldsymbol{m} \times \boldsymbol{B} \qquad \textbf{(34-2)}$$

d'Arsonval Meter Movement

A schematic of a d'Arsonval galvanometer is shown in Fig. 34-6. The torque on a loop in a magnetic field offset by the torque of a spiral spring as shown here is the basis for mechanical meter movements. Galvanometers are mid-reading meters. Current meters use parallel (with the coil) resistors to change the scale of the meter. Voltmeters use large series resistors to take the small currents necessary to activate the meter to measure voltage.

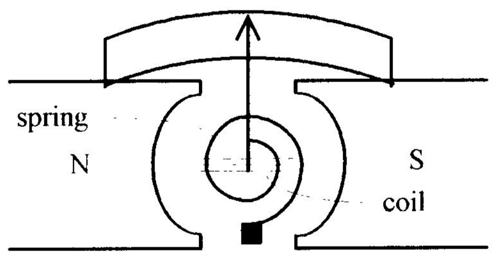

Fig. 34-6

The plane of the coil is parallel to the magnetic field so that a small current in the coil produces a torque on the coil which is offset by the torque of the spiral spring. With this arrangement, the rotation of the coil is proportional to the current. The external magnetic field (supplied by the permanent magnet) is curved (shaped) so the field is parallel to the plane of the coil through an angle of typically sixty degrees. Meter movements are rated as to how much current causes full scale deflection. A movement with a full scale deflection corresponding to $1.0\mu A$ is typical.

Energy Storage in Magnetic Materials

Magnetic dipoles placed in a magnetic field experience a torque. A simple model of a magnetic dipole is a molecule with a circulating current. These dipoles are rotated depending on the restoring torque of the material (structure). Picture a collection of magnetic dipolar molecules in a crystalline structure rotated by an external magnetic field thus stressing the structure. Consider the work done on a magnetic dipole by the application of an external magnetic field. This work is equivalent to the energy stored in the dipole-field combination.

C The energy of the system is taken as zero when ***m*** and ***B*** are at right angles ($\alpha = 90^o$). The energy at a position α, analogous to both the electrical and mechanical relationship between torque, angle, and energy, is

$$U = \int_{90^o}^{\alpha} \tau d\alpha = \int_{90^o}^{\alpha} IAB\sin\alpha d\alpha = -mB\cos\alpha\Big|_{90^o}^{\alpha} = -mB\cos\alpha$$

In vector notation

$$U = -\boldsymbol{m} \cdot \boldsymbol{B} \tag{34-3}$$